Fluid:-

- Fluid
  - liquid
    in compressible
    definite volume.
  - solids/gas.
    No volume but
    compressible
  - Vapour
    eg: steam.

mid-I

1. Briefly explain & derive expressions for
   a) surface tension & viscosity of fluid.
   b) capillarity & vapour pressure of a fluid.

2. Explain briefly on Newtonian fluids.
   a) Non newtonian b) vapour pressure.
   c) Calucukate the sp.wt, sp.mass, sp.vol, s.p.gr of a fluid having a volume of $6m^3$ and weight 44KN.

3. Derive the Equation for capillary rise & fall when a small glass tube is inserted in mercury.

4. Derive an expression for total pressure & centre of pressure for a vertical plane surface submerged in liquid.

5. Determine the total pressure & centre of pressure of a circular plate of dia 1.5m which is placed vertically in water in such way that the centre of plate is 3m below the free surface of water.

6. a) Define metacentre & meta centric height.
   b) what is the difference b/w U-tube manometer & inverted U-tube manometer & where are they used.

Assignment - 1

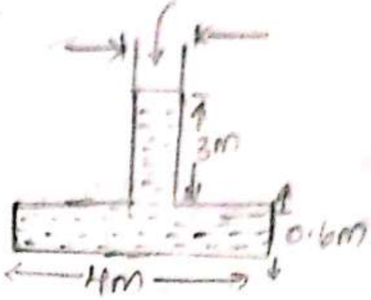
(1) How the viscosity varies with temperature for liquid and gases?

(2) Give a formula for total pressure and centre of pressure for a body? which is immersed in liquid at an inclination at θ as shown below.

(3) Following figure shows a tank of water find

a) i) Total pressure on the bottom of tank ii) weight of water in the tank and hydrostatic paradox b/w the results. Take width the tank as 2m.

b) what are Newtonian and non- newtonian fluids?



(1)(A) Temperature affects the viscosity. The viscosity of (gases) liquids decreases with the increase of temperature. while the viscosity of gases increases with increase of temperature. This is due to reason that the viscous forces in a fluid are due to cohesive and molecular momentum transfer. In the cohesive force predominates the molecular momentum transfer, due to closely packed molecules & with increase in temperature the cohesive decreases with result of decreasing viscosity. But in case of gases the cohesive forces are small molecular momentum transfer predominates, with increase in temperature. Molecular momentum transfer increases and hence increase viscosity.

(2)(A) Formula for Total pressure and centre of pressure of a body.

Total pressure $F = \rho g A \bar{h}$.

Centre of Pressure $= I_{G} = \frac{bd^3}{12}$.

3)a) (A) Given,

Depth of water on bottom of tank $h_1 = 3 + 0.6 = 3.6m$.

width of tank $= 2m$.

length of tank at bottom $= 4m$.

$\therefore$ Area at the bottom $= 4 \times 2$

$= 8m^2$.

i) Total pressure F, on the bottom is

$F = \rho g A \bar{h}$

$= 1000 \times 9.81 \times 8 \times 3.6$

$\boxed{\therefore F = 282528 N.}$

ii) weight of water in the tank.
$= \rho g \times$ Volume of tank.
$= 1000 \times 9.81 \times [3 \times 0.4 \times 2 + 4 \times 0.6 \times 2]$
$= 1000 \times 9.81 \times [2.4 + 4.8]$

$\boxed{\therefore w = 70632\,N.}$

iii) from i) & ii) it is observed that the total weight of water in the tank is much less than the total pressure at the bottom of the tank. This is known as "Hydrostatic pressure".

(5) (B)(A) <u>Newtonian law of fluids</u>:- It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the co-efficient of viscosity. $\tau = \mu \frac{du}{dy}$.

* Fluid which obey $\tau = \mu \frac{du}{dy}$ is called Newtonian law of fluids.

<u>Non-newtonian law of fluid</u>:-
The fluid which does not obey $\tau = \mu \frac{du}{dy}$ is called "Non-newtonian law of fluid".

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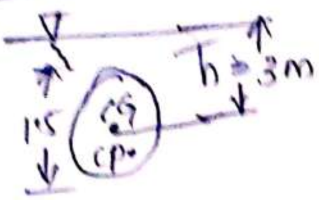
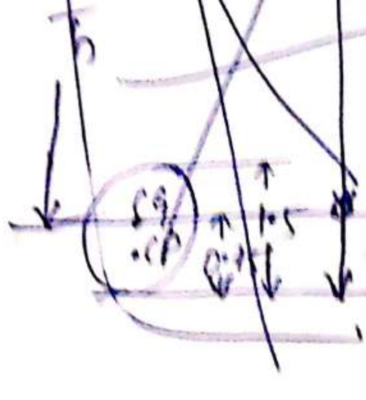
(1) Define meta centre.
(2) Express the velocity component u & v in terms of φ.
(3) Find the density of a metallic body which floats at the interface of mercury of specific gravity 13.6 & water such that 40% of its volume is sub-merged in mercury and 60% in water.

(1) (A) The theoretical point at which an imaginary vertical line passing through the centre of buoyancy and centre of gravity intersects the imaginary vertical line through a new centre of buoyancy created when the body is displaced or tipped in the water.

Sp. g of w = 1
den = 1000

(5) (A) Total Pressure on the Plate
$P = wA\bar{h}$
where w = Sp. wt of water = $9810\,N/m^3$
A = Area of Plate $= \frac{\pi}{4} d^2$
$= \frac{\pi}{4}(\quad)$





(5) (A) Given,
$d = 1.5m$, $\bar{h} = 3m$
Total pressure on the plate
$P = wA\bar{h}$
Sp. wt of water = $9810\,N/m^3$

$P = 9810 \times 1.76 \times 3 = 51796.8 N.$

Centre of Pressure : Depth of centre of pressure below the free surface is

$h^* = \bar{h} + \frac{I_G}{A\bar{h}}$ $(\because \bar{h} = 3m)$.

$I_G = \frac{\pi}{64} d^4 = \frac{\pi}{64} (1.5)^4 = 0.248\ m^4$

$h^* = 3 + \frac{0.248}{1.76 \times 3}$

$\boxed{h^* = 3.046m}$

Mid - II

1) a) Classify the different types of flows & explain in briefly.

b) The velocity potential function is given by $\phi = x^2 - y^2$. calculate the velocity component in x & y directions. Also. S.T. 'ϕ' represents a possible flow of fluid flow.

2) a) Define stream function b) Streak line c) free vortex flow d) forced vortex flow.

b) A horizontal Venturimeter with

3) c

3) a) Explain briefly the various forces acting on a fluid motion.

b) A pitot tube placed in the centre of 300mm pipe line

UNIT-I

Basic concepts & definitions.

Fluid:- A fluid is a substance which is capable of flowing or moving under the action of shear-force.

It has no definite shape but it conforms the shape of the containing vessel.

A fluid may be classified as * liquid
* solids/gas
* vapour.

Liquids:- Definite volume and incompressible

Gas :- It has no definite volume but compressible.

Vapour:- It is a gas whose temperature and pressure are very nearer to the liquid state.

Eg:- steam.

Ideal fluid:-

An ideal fluid is one which has no viscosity and surface tension and is incompressible. In true sense no such fluid existing nature. However fluids, which have low viscosity such as water and air can be treated as ideal fluids.

Real fluids:-

A real fluid is one which is having viscosity and surface tension and compressibility in addition to the density. The real fluids are available in nature.

Fluid mechanics:-

Fluid mechanics may be defined as the branch of science which deals with the behaviour of fluid under the conditions of rest & motion.

The fluid mechanics may be divided into '3' types they are

1. Fluid statics
2. fluid kinematics
3. fluid Dynamics.

Fluid statics:- Study of fluids at rest is called 'F.S'.

Fluid Kinematics :- Study of fluids in motion where pressure forces are not considered is called "F.K"

Fluid Dynamics:- Study of fluids in motion where pressure forces are considered.

Properties of fluids:-

The matter can be classified on the bases of spacing between the molecules of the matter as follows.

* fluid state * Solid state. * Gaseous state.

Solid state:- In Solid State molecules are very closely spaced where as in liquids the spacing between the different molecules is relatively large and in gaseous state the spacing between the molecules is still large. It means inter molecular cohesive-forces are large in solids. Small in liquids and extremely small is gases.

Properties:-

Density & mass density:- Density & mass density is defined as the ratio of the mass of a fluid to its volume. It is denoted by the symbol of "ρ".

S.I units is kg/m^3.

$$\boxed{\therefore \rho = \frac{\text{mass of a fluid}}{\text{volume of a fluid}} \ \frac{kg}{m^3}}$$

The volume of density of water is $1 gm/cm^3$ & $1000 kg/m^3$.

Specific weight & weight density:- It is defined as the ratio b/w the weight of a fluid to its volume. It is denoted by 'w'.

$$w = \frac{\text{weight of fluid}}{\text{volume of fluid}} \quad \& \quad \frac{\text{mass of fluid} \times \text{acceleration due to gravity}}{\text{volume of fluid}}$$

$$\boxed{w = \rho g \ N/m^3}$$

The volume of specific weight & weight density of water is $9.81 \times 1000 \ N/m^3$.

Specific volume:- specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass.

$$\text{Specific volume} = \frac{\text{volume of fluid}}{\text{mass of fluid}} = \frac{V}{m} = \frac{1}{\frac{\text{mass of fluid}}{\text{volume of fluid}}} = \frac{1}{\rho}$$

$$\boxed{\text{units:- } \frac{1}{\rho} \ m^3/kg}$$

Specific gravity:- It is the ratio of specific weight of fluid to the specific weight of standard fluid. It is denoted by 's'.

$$S = \frac{\text{Specific weight of fluid}}{\text{Specific weight of standard fluid}} \ (\&) \ \frac{\text{Density of fluid}}{\text{Density of standard fluid}}$$

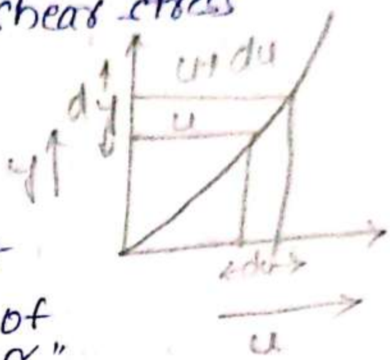
for liquids standard fluid is water.

for gases standard fluid is air.

It is a dimension less quantity. The problems are continued after 2 paper

viscosity:-

viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of a fluid. when two layers of a fluid is at a distance of dy apart move one over the other with different velocities u, du+u as shown in fig. The viscosity together with relative causes a shear stress acting b/w the fluid layers.



The top layer causes a shear stress on the adjacent lower layer "y" the upper layer causes a shear stress on the adjacent this shear stress is proportional to the rate of change of velocity w.r.t 'y'. It is denoted "τ".

$\tau \propto \frac{du}{dy}$ du = increased velocity.
dy = distance b/w two layers.

$\frac{du}{dy}$ = velocity gradient (or) rate of change of velocity.

$$\tau = \mu \frac{du}{dy} \quad \text{(1)}.$$

μ = co-efficient of dynamic viscosity or co-efficient of proportionality.

$\frac{du}{dy}$ = rate of shear strain.

from (1) $\mu = \dfrac{\tau}{\frac{du}{dy}}$

Thus, viscosity is also defined as the shear stress required to produce unit weight of shear strain.

Units of viscosity:-

$$\mu = \frac{\tau}{\frac{du}{dy}} = \frac{\text{force/area}}{\frac{\text{distance/time}}{\text{distance}}} = \frac{\text{force/area}}{\frac{\text{length}}{\text{time}} \times \frac{1}{\text{length}}} = \frac{\text{force/area}}{\text{Time}}$$

$= N\text{-}S/m^2$.

In M.K.S system = $\frac{kgf\text{-}sec}{m^2}$ → 1 kgf = 9.81 N.

CGS system = $\frac{dyne\text{-}sec}{cm^2}$ → 1 N = 10^5 dyne.

S.I system = poise → $\frac{1 kgf\ sec}{m^2}$ = 98 . poise

$\frac{1 N.S}{m^2}$ = 10 poise

1 centipoise $= \frac{1}{100}$ poise.

If the viscosity is given poise it must be divided by 100.

The viscosity of water at 20°c is 0.01 poise and 1.0 centipoise.

Kinematic viscosity:-

It is defined as the ratio between the dynamic viscosity and density of fluid. It is denoted 'μ' 'ν'

density = $\frac{mass}{volume}$.

$$\nu = \frac{\mu}{\rho}$$

The units of Kinematic viscosity.

(F = m.a)

$$\nu = \frac{force \times time}{(length)^2 \times \frac{mass}{(length)^3}} = \frac{Force \times time}{mass/length}$$

$$= \frac{mass \times acceleration \times time}{mass/length} = a \times time \times length.$$

$a = m/s^2$

$$\nu = \frac{length}{(time)^2} \times time \times length = \frac{l^2}{time}.$$

$$\boxed{\nu = m^2/sec.}$$

In C.G.S. unit kinematic viscosity is also known as stroke.

One stroke = $cm^2/sec = \frac{1}{(100)^2} m^2/sec.$

1 centi stroke = $\frac{1}{100}$ x stroke.

Newton's law of viscosity:-

It states that shear stress(τ) on a fluid element is directly proportional to the rate of shear strain. The constant of proportionality is called co-efficient of viscosity.

Mathematically $\tau \propto \frac{du}{dy}$ $\tau = \mu \frac{du}{dy}$

Newtonian fluid:- It obeys the newton's law of viscosity $\tau = \mu \frac{du}{dy}$

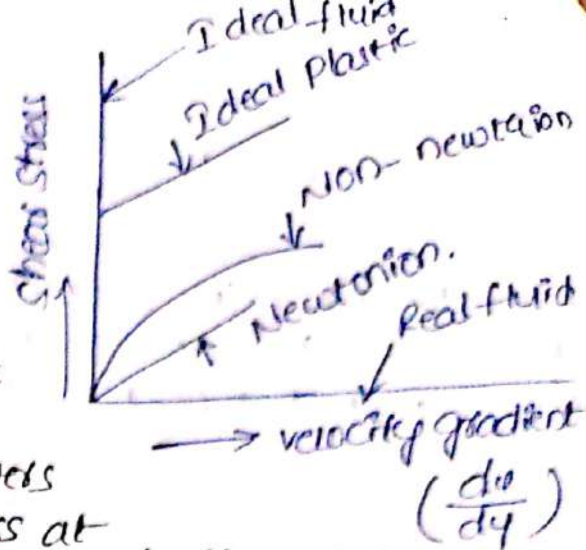
non-newtonian fluid:- It doesnot obey the newton's law of viscosity.

Types of fluids:-

* Ideal fluid
* Real fluid
* Newtonian fluid
* Non-newtonian fluid
* Ideal plastic fluid

Ideal Plastic Fluid.

Q-1



v.v.imp

If the velocity distribution over a plate is given by $u = \frac{2}{3}y - y^2$ in which u is the velocity in m/sec. at a distance 'y' meters above the plate. Determine shear stress at $y=0$ & $y=0.15$ m Take dynamic viscosity of a fluid as 8.63 poise.

Sol:- Given,

Velocity $u = \frac{2}{3}y - y^2$

τ at $y=0$, τ at $y=0.5$ $\quad \mu = 8.63$ poise $= \frac{8.63}{10}$ N-s/m²

$\frac{du}{dy} = \frac{2}{3}.(1) - 2y$ at $y=0$

$\frac{dy}{dy} = \frac{2}{3}. = 0.667$

$\left(\frac{du}{dy}\right)_{y=0.15} = \frac{2}{3} - 2y = \frac{2}{3} - 2(0.15)$

$= 0.367$

$\tau_{y=0} = \mu\left(\frac{du}{dy}\right)_{y=0}$

$= \frac{8.63}{10} \times 0.667 = 0.575$ N/m²

$\tau_{y=0.15} = \mu\left(\frac{dy}{dy}\right)_{y=0.15} = \frac{8.63}{10} \times 0.367$

$= 0.316$ N/m².

A Plate 0.025 mm distance from a fixed plate moves at 60 cm/sec and requires a force of 2N / unit area to maintain the speed. Determine the viscosity of fluid b/w the plates.

$du = 0.6 - 0 = 0.6$

$dy = 0.025 \times 10^{-3}$

$\tau = 2$ N/m².

Given,

$u = 60$ cm $= 0.6$ mm

$du = 0.6 - 0 = 0.6$

$dy = 0.025 \times 10^{-3}$ M

$\tau = 2$ N/m²

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$\tau = \mu \frac{du}{dy}$

$2 = \mu \left(\frac{0.6}{0.025 \times 10^{-3}} \right)$

$\mu = \frac{2}{24000}$

$\therefore \mu = 8.33 \times 10^{-5}\ N\text{-}S/m^2$.

Find the kinematic viscosity of an oil having density $981 kg/m^3$. The shear stress at a point in oil is $0.2452 N/m^2$ & velocity gradient at that point $0.2/Sec$

Given,

density of an oil $\rho = 981 kg/m^3$.

Shear stress of an oil $= 0.2452\ N/m^2 = \mu$

velocity gradient $\left(\frac{du}{dy}\right) = 0.2\ /Sec$

$\tau = \mu \times \frac{du}{dy} = 0.2452 \times 0.2$

$\tau = 0.049$

kinematic viscosity $(\nu) = \frac{\mu}{\rho} =$

$\nu = \frac{\mu}{\rho}$ $\frac{1.266}{981}$

V.V.imp

Determine the intensity of shear stress of an oil having viscosity 1poise the oil is used for lubricating the clearance b/w a shaft of diameter 10cm & its general bearing the clearance is 1.5mm & shaft rotates at 150rpm.

Sol: Given,

Diameter of shaft $= 10cm = 0.1m$

Speed of shaft $N = 150 rpm$

clearance $(dy) = 1.5mm = 0.0015m$

viscosity $\mu = 1 poise = \frac{1}{10} = 0.1 N\text{-}S/m^2$

Velocity of shaft $u = \boxed{\frac{\pi DN}{60}} = \frac{3.14 \times 0.1 \times 150}{60}$

$u = 0.785\ m/sec$

Change of velocity $= du = u_f - u_i = 0.785 - 0.$

$du = 0.785$

Shear stress $\tau = \mu \cdot \frac{du}{dy} = 0.1 \times \frac{0.785}{0.0015}$

$= 52.33\ N/m^2$

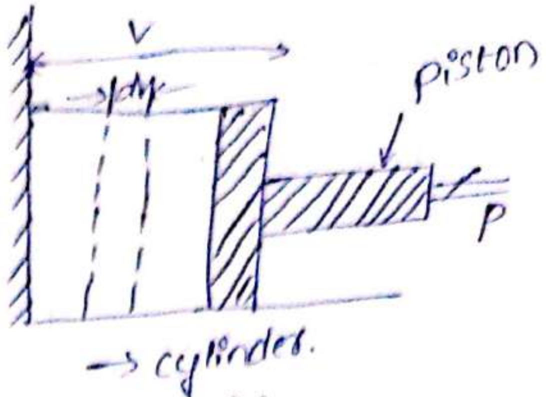
Compressibility & Bulk modulus:-

Bulk modulus:- It is the change in volume is known as Bulk modulus. 'K'

Compressibility:- It is the reciprocal of bulk modulus. "$\frac{1}{K}$"

Bulk modulus is denoted by 'K'

$$K = \frac{\text{Increase in pressure}}{\text{volumetric strain}}$$



Determine the bulk modulus of elasticity of a liquid if the pressure of the liquid is increased from 70 N/cm² to 130 N/cm². The volume of liquid decreases by 0.15%.

Sol:- Given,

Initial Pressure $(P_i) = 70\ N/cm^2 =$

Final pressure $(P_f) = 130\ N/cm^2$

volume of liquid $= 0.15\% = \frac{0.15}{100}$

$dp = P_F - P_i$

$= 130 - 70 = 60\ N/cm^2$

$$K = \frac{dp}{\frac{-dv}{v}} = \frac{60}{\frac{0.15}{100}} = 4 \times 10^4\ N/cm^2$$

Problems

① calculate specific weight, density and specific gravity of 1liter of a liquid which weights 7N.

Sol:- Given,

weight of fluid $= 7N$ and volume $=$ 1litre $= \frac{1}{1000}m^3 = 10^{-3}m^3$.

specific weight $w = \frac{\text{weight}}{\text{Volume}} = \frac{7}{10^{-3}}$

$w = 7 \times 10^3\ N/m^3$.

Density $\rho = \frac{w}{g} = \frac{7000}{9.81}$

$\rho = 713.5\ kg/m^3$.

Specific gravity $S = \frac{\text{specific weight of fluid}}{\text{Specific wt. of Standard fluid}} = \frac{7000}{9.81 \times 1000} = 0.7135$

(or)

$S = \frac{\text{wt. density of fluid}}{\text{wt. density of st. fluid}} = \frac{713.5}{1000}$

$= 0.7135$.

(2) Calculate the density, specific weight and weight of one litre of petrol of specific gravity is 0.7.

Sol: Given,

Volume = 1 litre = $10^{-3} m^3$; specific gravity $S = 0.7$

$$\text{Specific gravity} = \frac{\text{sp. wt of fluid}}{\text{sp. wt of st. fluid}}$$

$$0.7 = \frac{\text{sp. wt of fluid}}{9.81 \times 1000}$$

Sp. wt of fluid $w = 0.7 \times 9.81 \times 1000$.

$$\boxed{w = 68.67}$$

$$\text{Sp. wt. } w = \frac{\text{wt. of fluid}}{\text{volume of fluid}} \Rightarrow 68.67 = \frac{\text{wt. of fluid}}{10^{-3}}.$$

$w = 6867 \times 10^{-3} N$

$$\text{Density } \rho = \frac{w}{g} = \frac{6867}{9.81}$$

$\rho = 700 kg/m^3$.

(3) Calculate the sp. weight, sp. mass, sp. volume and sp. gravity of liquid having a volume of $6m^3$ and weight 44 KN.

Sol: Given,

Volume = $6m^3$; weight = 44 KN = $44 \times 10^3 N$.

$$\text{sp. weight } w = \frac{\text{wt. of fluid}}{\text{volume}} = \frac{44 \times 10^3}{6}$$

$w = 7333.33 N/m^3$ or $7.3333 KN/m^3$.

$$\text{Density } \rho = \frac{w}{g} = \frac{7333.33}{9.81}$$

$\rho = 747.53 kg/m^3$.

$$\text{Sp. volume} = \frac{1}{\rho} = \frac{1}{747.53}$$

$v = 1.337 \times 10^{-3} m^3/kg$ or $v = 0.00134 m^3/kg$.

$$\text{Sp. gravity} = \frac{\text{Sp. wt of fluid}}{\text{Sp. wt of standard fluid}}$$

$$= \frac{7333.33}{9.81 \times 1000}$$

$S = 0.747$.

Problems:-

1) Determine the bulk modulus of elasticity of a liquid, if the Pressure of the liquid is increased to N/cm² to 130 N/cm². The volume of the liquid decreases by 0.15%.

Given data:

Initial Pressure $(P_i) = 70\,N/cm^2$; final Pressure $(P_f) = 130\,N/cm^2$.

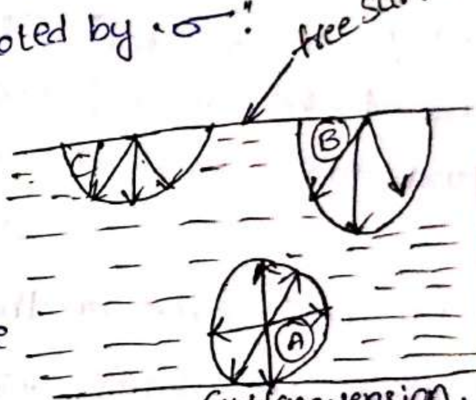
$dp = P_f - P_i = 130 - 70 = 60\,N/cm^2$

$-\frac{dv}{v} = 0.15\% = \frac{0.15}{100}$

$\therefore$ Bulk Modulus $k = \frac{dp}{\left(-\frac{dv}{v}\right)} = \frac{60}{0.15/100}$

$$\boxed{K = 4 \times 10^4\ N/cm^2}$$

Surface tension:- It is also defined as the tensile force per unit area. Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas (or) on the surface between the two immiscible liquids such that the contact surface behaves like a membrane under tension. It is denoted by 'σ'.



Surface tension.

Consider the molecules A, B, C of a liquid in a mass of a liquid the molecules 'A' is attracted in all directions equally by the surrounding molecules of the liquid. Then the resultant force acting on a molecule is zero. But the molecule 'B' which situated near the free surface is acted up on by upward and downward forces acting un-balanced. Thus the resultant force of molecule 'B' i.e. acting in the downward direction. The molecule 'C' situated on the free surface of liquid thus experience 'A' resultant downward force. All the molecules on the free surface experience a downward force then the free surfaces of liquid acts a very thin film under tension of the surface of the liquid act as though it is in elastic membrane under tension.

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Surface-tension on a liquid droplet:-

Consider a small spherical droplet of a liquid of radius 'r'. On the entaire surface of the droplet then tensile force due to surface tension will be acting.

* Soap bubble
* water drop
* water jet

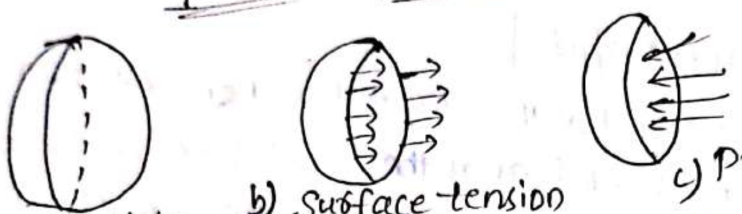
Let

σ = Surface tension of the liquid.

P = Pressure intensity inside the droplet.

d = Diameter of droplet.

Forces on droplet:-



a) Droplet b) Surface tension c) Pressure force.

Let the droplet is cut into two halves then the force acting on one half will be tensile force due to surface-tension acting around the circumference of the cut portion as shown in fig: (b) and this is equal to 'σ x circumference'

$$\Rightarrow \sigma \times \pi d$$

Pressure force on the area $= \pi/4 \times d^2 \times p$ as shown in fig c'.

These two forces will be equal and opposite under equilibrium conditions that is.

$$P \times \frac{\pi}{4} \times d^2 = \sigma \times \pi d$$

$$\frac{P.d}{4} = \sigma$$

$$\boxed{P = \frac{4\sigma}{d}} \qquad \sigma = \frac{Pd}{4} \longrightarrow \text{Surface tension.}$$

Surface tension on a hallow bubble:-

A hallow bubble is like a soap bubble in air has two surfaces on contact with air one is inside and other is outside. They the surfaces are subjected to surface-tension in such case we have.

$$P \times \frac{\pi}{4} \times d^2 = 2(\sigma \times \pi d)$$

$$\boxed{P = \frac{8\sigma}{d}} \qquad \sigma = \frac{Pd}{8}$$

Surface tension on a liquid jet:-

consider a liquid jet of diameter 'd' and length 'l' as shown in fig.

Let

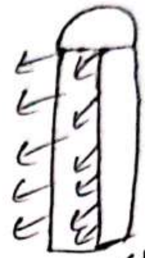
P- Pressure intensity inside

The liquid jet above the outside pressure.

σ = surface tension of the liquid.

consider the equilibrium of the semi jet we have

(a)



(b)

force due to pressure = P × area of semi jet

= P × L × d.

force due to surface tension = σ × 2L

$$P \times L \times d = \sigma \times 2L$$

$$Pd = 2\sigma$$

$$\boxed{P = \frac{2\sigma}{d}}$$

1) The surface tension of water in contact with air at 20°c is 0.0725 N/m The pressure inside a droplet water is to be 0.02 N/cm² greater than the outer pressure. Calculate the diameter of the droplet of water

Given data:

Surface tension σ = 0.0725 N/m.

Pressure intensity $P = 0.02 \; N/cm^2 = 0.02 \times 10^4 \; N/m^2$

Dia of droplet d = ?

Pressure intensity $P = \frac{4\sigma}{d}$.

$$d = \frac{4\sigma}{P} \rightarrow d = \frac{4 \times 0.0725}{0.02 \times 10^4}.$$

$$\boxed{d = 1.45 \times 10^{-3} \; M}$$

2) find the surface tension in a soap bubble of 40mm dia. when the inside pressure is 2.5 N/m² above atmospheric pressure

Given data

Pressure $P = 2.5 \; N/m^2$

Dia d = 40mm = 40×10^{-3} m.

$\therefore$ Pressure intensity $p = \frac{8\sigma}{d}$

$$2.5 = \frac{8 \times \sigma}{40 \times 10^{-3}}$$

$$\sigma = \frac{2.5 \times 40 \times 10^{-3}}{8}$$

$$\boxed{\sigma = 0.0125\ N/m}$$

3) The pressure outside the droplet of water of diameter 0.04mm is 10.32 N/cm^2. Calculate the pressure within the droplet if surface tension is given as ~~0.075~~ 0.0725 N/m at water.

Given data:

diameter $d = 0.04mm = 0.04 \times 10^{-3} m$.

outside pressure $= 10.32\ N/cm^2$.

Surface tension $\sigma = 0.0725\ N/m$.

Pressure intensity $p = \frac{4\sigma}{d}$

$$p = \frac{4 \times 0.0725}{0.04 \times 10^{-3}}$$

$$p = 7250\ N/m = 0.725\ N/cm^2.$$

$\therefore$ Inside pressure on droplet $= p +$ Pressure outside droplet

$$= 0.725 + 10.32$$

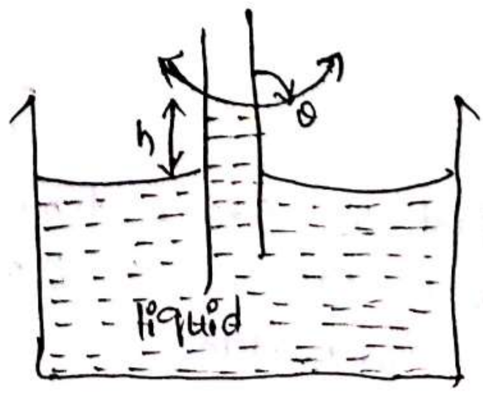
$$= 11.045\ N/cm^2.$$

Capillarity:-

Capillarity is defined as phenomenon of rise (or) fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. The rise of the liquid is known as "capillary rise". While the fall of the liquid surface is known as "capillary fall expression".

Expression for capillary rise:-

Consider a glass tube of small diameter opened at both ends and is inserted in a liquid say water. The liquid will rise in the tube above the level of the liquid.



let h be height of the liquid in the tube.

Under a static equilibrium the weight of liquid of height 'h'. is bounded by the force at but the force at the surface of the liquid in the tube is due to surface tension

Let σ = surface tension of liquid.

θ = angle of contact between liquid & glass tube.

The weight of liquid height 'h' = (Area of tube × h) × e × g

$$= \left(\frac{\pi}{4}d^2 \times h\right) \times e \times g \longrightarrow (1)$$

Where

e = density of liquid.

vertical compound of the surface tensile force

$$= \sigma \times \text{circumference} \times \cos\theta$$

$$= \sigma \times \pi d \times \cos\theta \longrightarrow (2)$$

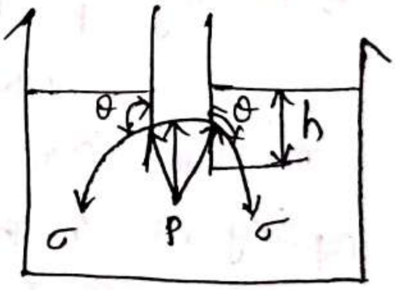
Equ (1) & (2).

$$\frac{\pi}{4} \times d^2 \times h \times e \times g = \sigma \times \pi d \times \cos\theta$$

V.V.imp. $h = \dfrac{4\sigma\cos\theta}{egd}$

but $\boxed{h = \dfrac{4\sigma}{egd}}$

Expression for Capillary fall:-



If the glass tube is dipped in mercury. the level of mercury in tube will be lower than the general level of the outside liquid as shown.

Let h = height of depression in tube.

Then in equilibrium two forces are acting on the mercury inside the tube. first one is due to surface tension acting in downward.

$$\sigma \times \pi d \times \cos\theta \longrightarrow (1)$$

Second force is due to hydrostatic force acting upward and is equal to intensity of pressure at a depth h × area

$$\text{Area} \times h \times P \longrightarrow (2) \qquad \because (P = eg)$$

Equating the equ (1) & (2).

we get $\boxed{h = \frac{4\sigma\cos\theta}{\rho g d}}$

Value of 'θ' for mercury and glass tube is 130°.

---

1) Calculate the capillary rise in a glass tube of 2.5mm diameter immerse the vertically in (a) water (b) Mercury. Take surface tension $\sigma = 0.0725$ N/m for water and $\sigma = 0.52$ N/m for mercury in contact with air. The 'p' gravity for mercury is given as 13.6 and angle of contact 130°.

Sol: Given data:-

Diameter of tube $d = 2.5$ mm $= 0.0025$ m $= 2.5 \times 10^{-3}$ m.

Surface tension for water $= 0.0725$ N/m.

Surface tension for mercury $= 0.52$ N/m.

S.p gravity for mercury $(g) = 13.6$

Angle $\theta = 130°$

a) Capillary rise in water:-

$$h = \frac{4 \times 0.0725}{1000 \times 9.81 \times 2.5 \times 10^{-3}} = 0.0118 \text{ m}.$$

b) Mercury:-

$$h = \frac{4 \times 0.52 \times \cos 130°}{9.81 \times 13.6 \times 1000 \times 2.5 \times 10^{-3}} = -4.008 \text{ m}.$$

The '-ve' sign are capillary fall (or) pressure.

2) The capillary rise in the glass tube is not to exceed 0.2mm of water. determine mine size given that surface tension for water in contact with air 0.0725 N/m.

Sol: Given data

height of capillary tube $h = 0.2$ mm $= 0.2 \times 10^{-3}$ M

Surface tension $\sigma = 0.0725$ N/m

$$h = \frac{4\sigma}{\rho g d}$$

$$0.2 \times 10^{-3} = \frac{4 \times 0.0725}{1000 \times 9.81 \times d} \Rightarrow d = \frac{4 \times 0.0725}{1000 \times 9.81 \times 2 \times 10^{-3}}$$

$$\boxed{d = 0.147 \text{ M}}$$

3) An oil
and S
Calculat
thicknes
Given d

5) An oil of velocity 5 poise is used for lubrication between a shaft and sleeve. The diameter of the shaft is 0.5 M and it rotate 200 rpm. Calculate the power lost in oil for a sleeve length of 100mm. The thickness of oil film is 1.0mm.

Given data:

viscosity $\mu = 5$ poise $= 0.5\ N-s/m^2$

Diameter of shaft $D = 0.5\ m$

Speed of shaft $(N) = 200$ rpm

Sleeve length $= 100mm = 0.1\ m$

Thickness $= 1mm = 1\times10^{-3} m$

tangential velocity of shaft $u = \frac{\pi D N}{60}$

$$= \frac{\pi \times 0.5 \times 200}{60} = 5.235\ M/sec.$$

Using the relation $\tau = \mu \frac{du}{dy}$

Where du = Changing velocity

$du = u - 0$

$= 5.235 - 0$

$du = 5.235\ M/Sec$

dy = Change of distance.

$dy = 1\times10^{-3}\ M$

This is the shear stress on the shaft

$\therefore$ Shear force on the shaft (F) = Shear stress x Area

$= \tau \times$ Area

$= \mu \times \frac{du}{dy} \times$ Area

$= 0.5 \times \frac{5.235}{1\times10^{-3}} \times \pi D \times L$

$= 0.5 \times \frac{5.235}{1\times10^{-3}} \times \pi \times 0.5 \times 0.1$

$F = 411.15\ N.$

Torque on the shaft (T) = force $\times \frac{D}{2}$

$= 411.15 \times \frac{0.5}{2}$

$= 102.7\ N\text{-}m.$

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Power lost $P_2 = T \times \omega$ watts

$$= 102.7 \times \frac{2\pi N}{60}$$

$$= 102.7 \times \frac{2\pi \times 200}{6}$$

$$P_2 = 2152 \text{ watts} = 2.15 \text{ kW.}$$

Date :- 28-12-2020.

Vapour Pressure :- The change from liquid state to the gaseous state is known as "vapourisation". The vapourisation occurs because of continous escape of molecules free liquid surface. consider a liquid like water which is confined in a closed vessale. let the temperature of liquid is 20°C and pressure is atmp. This liquid will vapourize at 100°C when vapourisation takes place the molecules get accumulated in the escape from the free surface of the

These vapour molecules get accumulated in the space b/w the free surface & the top of vessale. These accumulated vapour exert a pressure of liquid surface. This pressure is known as 'vapour pressure' of the liquid (or). This is the pressure at which the liquid is converted into vapours.

Fluid Pressure at a point :-

consider a small area dA in large mass of fluid. If the fluid is stationary, then the force exerted by the surrounding fluid on the dA is always perpendicular to the surface dA. let dF is the force acting on the area dA in the normal direction. Then the ratio of $\frac{dF}{dA}$ is known as intensity of pressure (or) simply pressure and this ratio is represented by 'p'. Mathematically the pressure at a point in a fluid at rest is $p = \frac{dF}{dA}$. If the force is uniformly distributed over the area then the pressure at any point is given by $p = \frac{\text{Force}}{\text{area}}$

Force = p × Area.

The units of pressure are kgf/m^2 or kgf/cm^2 in M.K.S. units.

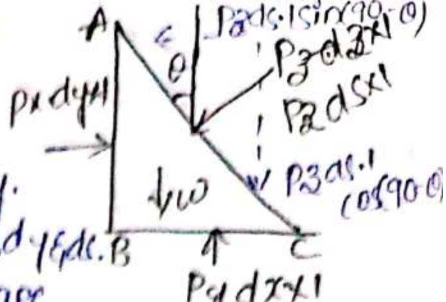
N/m^2 (or) N/mm^2 in S.I units.

N/m^2 also known as pascal.

Other commonly used units are $kPa = 1000 N/m^2$.

$bar = 100 kPa = 10^5 N/m^2$.

Pascal's law:- It state that the pressure acting on a fluid is equal in all directions. This is proved as



let Px, Py, P3 pressure forces acting on a body.
The fluid element is of very small direction dx, dy & ds.
Consider an orbitary fluid element of vedge shape in a fluid mass at rest as shown in fig. let, width of element ⊥er to the paper is unity (1). Px, Py, P3 pressure acting on the face AB, BC, CA. Angle BAC = θ. There are two types of forces they are 1. pressure forces normal to the surface.
2. weight of element in the vertical direction.

The forces on faces $AB = P_x dy \cdot 1$.
force on face $BC = P_y dx \cdot 1$
force on face $AC = P_3 ds \cdot 1$

[weight = $\rho \times g$]

[mass = density × volume
$\rho \times V$]

weight of the element (ω) = mass × gravity.
$= \rho \times v \times g$

Resolving the forces in 'x' direction.

$P_x dy \cdot 1 - P_3 ds \cdot 1 \sin(90-\theta) = 0$.

$P_x dy 1 = P_3 ds \cdot 1 \cos\theta = 0 \Rightarrow P_x$

from fig $\cos\theta = \frac{adj}{hy} = \frac{dy}{ds}$

$P_x dy = P_3 ds\left(\frac{dy}{ds}\right)$

$P_x = P_3$ ——— (1)

Resolving the force in 'y' direction

$P_y dx \cdot 1 = P_3 ds \cdot 1 \sin\theta - \rho v g$.

$P_y dx \cdot 1 - P_3 ds \cdot 1 \sin\theta - \rho g\left(\frac{dx \cdot d}{2}\right)$

As the element is very small so wt of the element is neligible.

$P_y dx \cdot 1 - P_z ds \cdot 1 \cdot \frac{dx}{ds} = 0$.

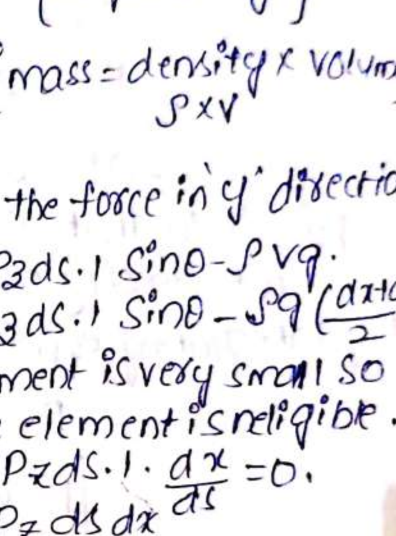
$P_y dx = P_z ds \frac{dx}{ds}$

$P_y = P_z$ ——— (2)

from (1) & (2)
The pressure at any point in x, y & 3 directions are equal.

$$\boxed{\therefore P_x = P_y = P_3}$$

Hydrostatic law:- The pressure at any point in a fluid at rest is obtained by hydrostatic law which states that rate increased in pressure in vertical direction must be equal to the sp. wt of a fluid at that point.



Consider a small fluid element as shown in fig. let, ΔA = cross-sectional area of the element
ΔZ = height of fluid element
P = Pressure on face 'AB'.
Z = distance of fluid element from free surface.

The forces acting on the fluid element are
Pressure force on AB = $P \times \Delta A$ acting per to the face AB in ↓ direction
Pressure force on CD = $(P + \frac{\partial P}{\partial z} \times \Delta z)\Delta A$ acting per in upward direction.
weight of fluid element = $\rho g v$.
$= \rho g(\Delta A \times \Delta z)$.
Pressure forces on surface BC & AD are quit opposite.

for equilibrium of fluid element

$P \times \Delta A - (P + \frac{\partial P}{\partial z}\Delta z)\Delta A - \rho g v = 0$.

$P \times \Delta A - P\Delta A - \frac{\partial P}{\partial z}\Delta A \Delta z - \rho g(\Delta A \times \Delta z) = 0$

$-\frac{\partial P}{\partial z}\Delta A \Delta z = \rho g(\Delta A \times \Delta z)$

$-\frac{\partial P}{\partial z} = \rho g$ — ① $\quad w = -\frac{\partial P}{\partial z}$

$-\partial P = \rho g \, \partial z$

Eq ① States that rate of increase in pressure in vertic =
wt. density of fluid at that point. This is hydro static law.

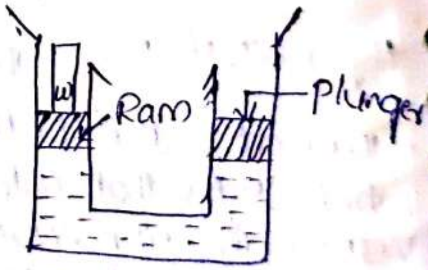
By integrating Eqn ① $\int \partial P = \int \rho g \, \partial z$

$P = \rho g z$, $\quad z = \frac{P}{\rho g} \quad z =$ pressure head.

$\boxed{P = wz}$

A hydraulic pressure has ram of 30 cm dia & a plunger of 4.5 cm dia meter. Find the weight lifted by the hydrolaic pressure when the force applied at the plunger is 500N.

Sol: Given,



Diameter of ram = 30cm = 0.3M
Diameter of Plunger = 4.5cm = 0.045M
Force in Plunger = 500N.
weight lifted by hydroulic pressure = W = ?

∴ Area of ram = $\frac{\pi}{4} \times D^2 = \frac{\pi}{4}(0.3)^2 = 0.07068 m^2$.

Area of Plunger = $\frac{\pi}{4} \times D^2 = \frac{\pi}{4} \times (0.045)^2 = 0.00519 m^2$

∴ Pressure intensity of Plunger = $\frac{Force}{area} = \frac{500}{0.00519} = 314465.408 N/m^2$

Due to Pascal's law the Pressure intensity in all directions are same.

Pressure intensity at ram = $\frac{W}{Area\ of\ ram}$

$314465.408 = \frac{W}{0.07068}$

$w = 22.226 \text{ kN}$

Calculate the pressure due to a column 0.3 M a) water b) Oil c) $\rightarrow 0.85 \rho g$ mercury of sp.g. 13.6 take density of water $\rho = 1000 \text{ kg/m}^3$.

Sol: Given,

Height of liquid column = 0.3 M

Specific

The pressure at any point in fluid is given by.

$P = \rho g z$

a) water

$P = \rho g z$

$P = 1000 \times 9.81 \times 0.3$

$= 2943 \text{ N/m}^2$

b) Oil

$P = \rho g z$

Sp gravity of Oil = 0.8

$$\text{Sp. gravity} = \frac{\text{wt of fluid}}{\text{wt of stan. fluid}}$$

$$0.8 = \frac{\text{wt of a fluid}}{1000}$$

$0.8 \times 1000 = \text{weight}$

$\rho = 800 \text{ kg/m}^3$

$P = \rho g z$

$P = 800 \times 9.81 \times 0.3$

$P = 2354.4 \text{ N/m}^2$.

c) Mercury

Sp. gravity = 13.6

$$\text{Sp. gr} = \frac{\text{wt of fluid}}{\text{wt. of stand fluid}}$$

weight = 13600 kg/m^3.

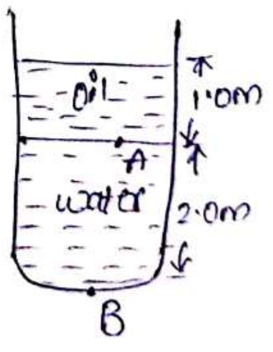
$P = \rho g z$

$P = 13600 \times 981 \times 0.3$

40024 N/m^2.

An open tank contains water upto a depth of 2m & above it an oil at 1m depth. i) At the interface of the two liquids ii) Bottom of tank. Find the pressure intensity at any point for above mention data. Sp. gravity of oil 0.9

Sol: Given



Height of water $z_1 = 2\text{M}$

Height of Oil $z_2 = 1\text{M}$

Sp gravity of oil = 0.9.

Density of oil $\rho_o = 0.9 \times 1000$
$= 900 \text{ kg/m}^3$.

Density of water $\rho_1 = 1000 \text{ kg/m}^3$.

i) At the interface of two liquids.

Pressure intensity $P_o = \rho_o g z$

$= 900 \times 9.81 \times 1.0$.

$P_o = 8829 \text{ N/m}^2$.

ii) At the Bottom

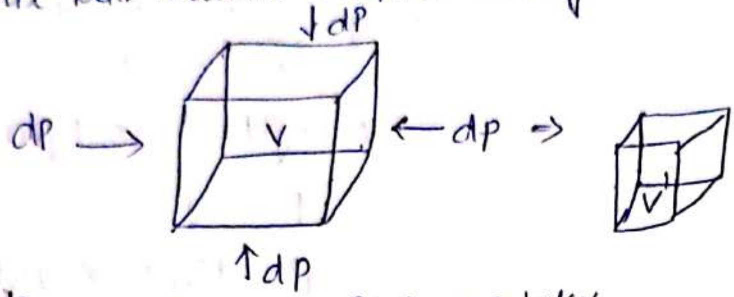
$$p = \rho_1 g z_1 + \rho_2 g z_2$$

$$p = 1000 \times 9.81 \times 2 + 900 \times 9.81 \times 1.$$

$$p = 28449 \ N/m^2.$$

Bulk modulus:

It is defined as the pressure required to cause a unit change of volume of a liquid. Since most liquids are practically incompressible, they require very large pressure to cause any significant volume change. For most liquids, the bulk modulus is approximately in the 250,000 - 300,000 psi.



$$K = -V \frac{dp}{dv}$$

K = Bulk modulus.

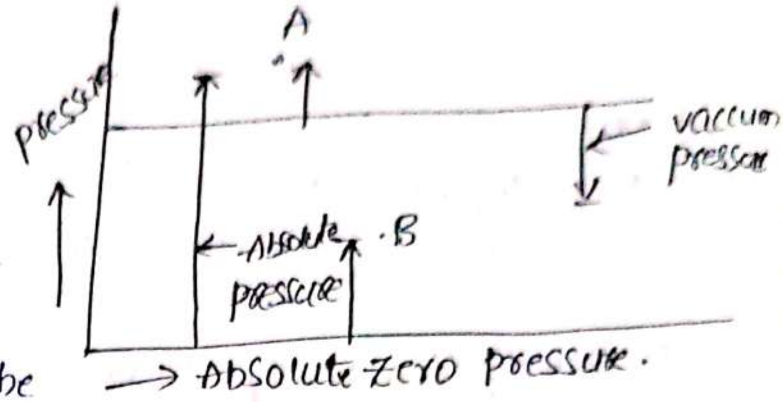
V = initial volume of the substance.

p = pressure.

UNIT-II.

Date:- 30-12-20

Absolute, Gauge, Atmospheric & Vaccum Pressure:-



The Pressure is measured in two different system. In one system it is measured above the absolute zero (or) complete vaccum & it is called absolute pressure and in other system pressure is measured above the atmospheric & it is called gauge pressure.

Absolute Pressure:-

If pressure intensity is expressed with respect to absolute zero is also called as absolute pressure.

Atmospheric Pressure:-

A normal pressure expected by atmospheric pressure on all surface be which it is in contact is known "Atmospheric pressure". At mean sea level and at 15°C atmospheric pressure is $10.104 \times 10^4 N/m^2$.

Gauge Pressure:-

If the absolute pressure intensity at a point is greater than the local atmospheric pressure the difference of this two pressures is called "Gauge Pressure".

Gauge Pressure = (Absolute − Atmospheric) pressure.

Vaccum pressure:-

If the absolute pressure intensity at a point is less than the local atmospheric pressure the difference of this two pressure is called "vaccum pressure"

Vaccum pressure = Atmospheric pressure − Absolute pressure.

Note:-

* The atmospheric pressure at sea level 15°c is $101.3\ KN/m^2$ & $10.13 N/cm^2$.
* The atmospheric pressure head is 760 mm of mercury & 10.33 meters of water.

What are gauge, absolute pressure at a point 3 meters below the free surface of a liquid having a density of 1.53×10^3 kg/m^3. If the atmospheric pressure is equivalent to 750 mm of mercury. The specific gravity of mercury is 13.6 & density of water is 1000 kg/m^3

Sol: Given,

Depth $z = 3$m

Density of liquid $\rho_1 = 1.53\times10^3$

Atmospheric pressure $z_0 = 750$ mm of mercury

$= 0.75$ m

Sp. gravity of mercury $= 13.6$.

Density of water $= 1000$

Sp. wt of fluid $\rho =$ sp. gravity of mercury × sp. gravity of water standard fluid

$= 13.6\times1000$

$\rho = 13600$ kg/m^3.

Atmospheric pressure $= \rho\times g\times z_0$

$= 13600\times9.81\times0.75$

$= 100062$ N/m^2.

Gauge pressure $= \rho_1 g z$

$= 1.53\times10^3\times9.81\times3.$

$= 45027.9$ N/m^2

Atmospheric pressure $= 100062 + 45027.9$

$= 145089.9$ N/m^2.

Measurement of Pressure:- The pressure of the fluid is measuring (piezometer) by using the following devices.

1. Manometer. 2. Mechanical Gauge.

1. Manometer:- Manometer are defined as the devices used for measuring the pressure at a point of a fluid by ballancing the column of fluid by the same (or) another column of fluid. They are classified as

a) Simple manometer b) Differential manometer.

2. Mechanical Gauges:- Mechanical guages are defined as devices used for measuring the pressure by ballancing the fluid column by the spring (or) dead weight. The commonly used mechanical pressure gauges are

a) Diaphram pressure Gauge b) Bourden tube pressure Gauge

c) Dead weight pressure Gauge d) Bellows pressure guage.

Vaccum Pressure:

For measuring vaccum pressure the level of heavy liquid in the manometer will be shown in the fig. then the pressure above datum line in the left column $= \rho_2 g h_2 + \rho_1 g h_1 + P$

The pressure head in the right column above datum line $= 0$ then equating two equations.

$$\rho_2 g h_2 + \rho_1 g h_1 + P = 0.$$

$$\boxed{P = -(\rho_2 g h_2 + \rho_1 g h_1).}$$

The right limb of a simple U-tube manometer contain mercury is open to the atmosphere. while the left limb is connected to the pipe in which a fluid of sp.gravity 0.9 is flowing. The centre of the pipe 12cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the 2 limbs is 20cm.

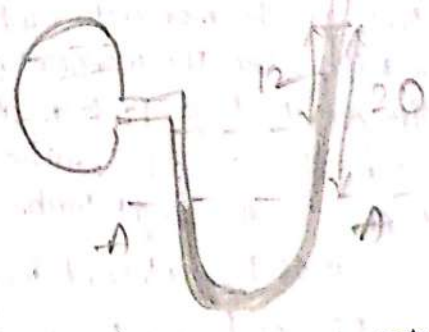
Sol:- Given,

Sp. gravity of liquid $= 0.9$

Density of liquid $\rho_1 = 1000 \times 0.9$
$= 900\ kg/m^3$.

Sp. gravity of mercury $S_2 = 13.6$.

Density of liquid $\rho_2 = 13600\ kg/m^3$.



density $= 1000 \times$ gravity

$h_2 = 20cm = 0.2m$

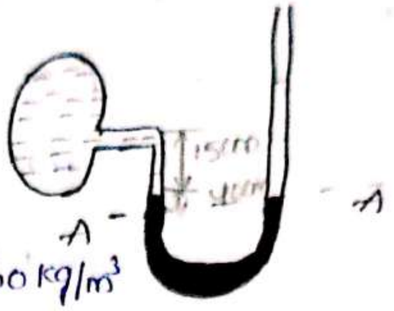
$h_1 = 20 - 12 = 8cm = 0.08m$.

$P = \rho_2 g h_2 - \rho_1 g h_1$

$=$

A simple U-tube manometer contain mercury is connected to a pipe in which fluid of sp.g 0.8 & having vaccum pressure is flowing on the other end of manometer is open to the atmosphere find the vaccum pressure in which difference of mercury in 2 limbs is 40cm & height of liquid in the left limb from the centre of pipe is 15cm.

Sol: Given



sp. gravity of fluid $= 0.8$

Density $\rho_1 = 0.8 \times 1000 = 800.$

sp gravity of mercury $S_2 = 13.6$

Density of mercury $\rho_2 = 13.6 \times 1000 = 1360\ kg/m^3$

$h_1 = 15cm = 0.15m,\ h_2 = 40cm = 0.4m$

$\therefore$ Vaccum pressure $P = -(\rho_2 g h_2 + \rho_1 g h_1) = -(13600 \times 9.81 \times 0.4 + 800 \times 9.81 \times 0.15)$

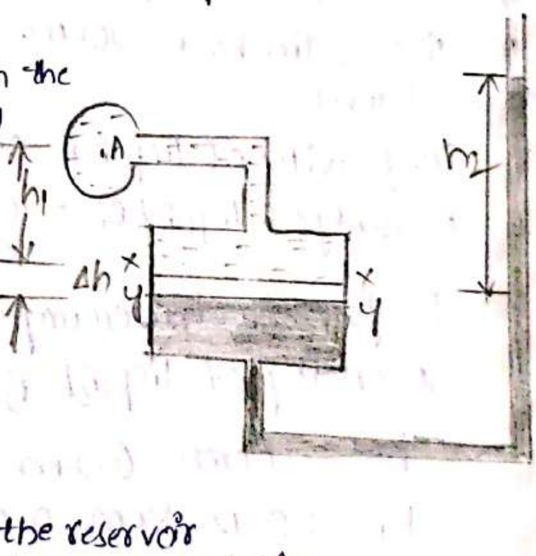
$= -54543.6\ N/m^2$ (or) $-5.4543\ N/cm^2.$

iii) Single column manometer:

Single column manometer is a modified form of a U-tube manometer in which a reservior having a large area (100 times) as compared to the area of the tube is connected to one of the limb (left limb) of the manometer as shown in fig. due to large c/s area of the reservoir for any variation in pressure, the change in liquid level in the reservoir will be very small which may be neglected. There are two types of manometer.

1. Vertical single column manometer. 2. Inclined single column manometer.

1. Vertical Single column manometer:-



let x-x be the datum line in the reservoir & in the right limb of the manometer. when it is not connected to the pipe when the manometer is connected to the pipe due to high pressure at 'A'. The heavy liquid in the reservior will be pushed downwards & will rise in the right limb.

Δh = fall of heavy liquid in reservoir.

h_2 = Rise of heavy liquid in right limb.

h_1 = height of centre of pipe above x-x.

P_A = pressure at 'A'. A = c/s area of the reservoir

a = c/s area of the right limb, ρ_1 = Density of liquid in pipe

ρ_2 = Density of liquid in reservoir.

Fall of heavy liquid in reservoir will cause a rise of heavy liquid level in the right limb is $A \times \Delta h = a \times h_2 \Rightarrow \Delta h = \frac{a \times h_2}{A}$ —— (1)

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1.a) Simple manometer:

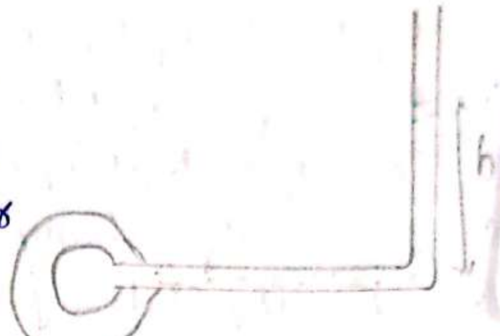
A simple manometer consists of a glass tube having one end is fixed to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are

i. Piezo meter.

ii. U-tube manometer.

iii. Single column manometer.

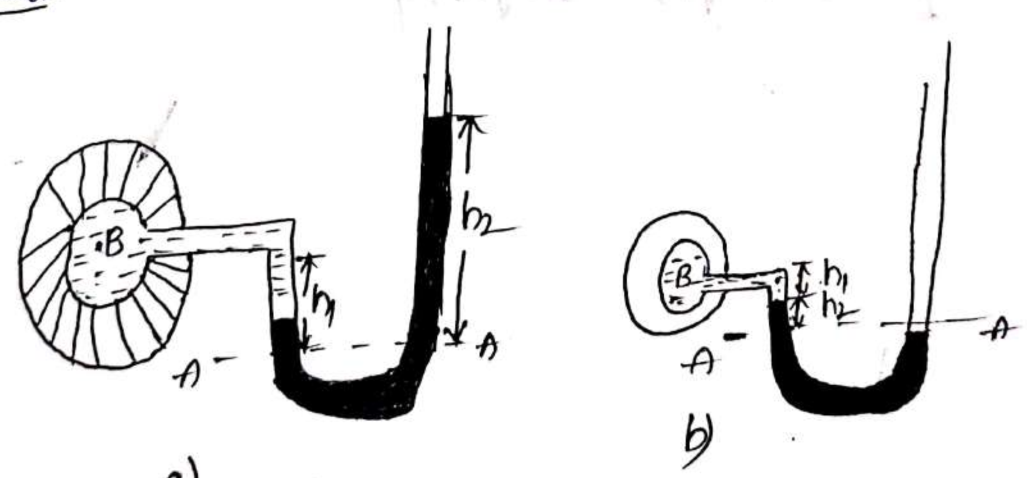
i. Piezometer:- It is the simplest form of manometer used for measuring guage pressure. One end of the manometer is connected to the point where pressure is to be measured and the other end is open to the atmosphere as shown in fig.



The rise of liquid gives the pressure head at that point. If at a point 'A' the height of liquid say water is 'h' meters in piezometer tube, then pressure at 'A' is equal to $\rho \times g \times h$.

ii) U-tube manometer:-

It contains of a glass tube bent in U-shape one end of which is connected to the point at which pressure is to be measured and other end of the tube remains open to the atmosphere as shown in fig. The tube generally contains mercury (or) any other liquid whose sp. gravity is grater than the sp. gravity of liquid.



U-tube manometer.

By using U-tube manometer we can measure gauge pressure & vaccum pressure.

-for gauge pressure:

let, 'B' is the point at which pressure is measured whose value is 'p'. The detum line 'A-A'.

h_1 = height of light liquid above the datum line.

h_2 = height of heavy liquid above the datum line.

S_1 = sp. gravity of light liquid.

ρ_1 = Density of light liquid = $1000 \times S_1$

S_2 = sp. gravity of heavy liquid

ρ_2 = Density of heavy liquid = $1000 \times S_2$

'A,' the pressure is same for the horizontal surface. Hence pressure above the horizontal datum line 'A-A' in the left column and in the right column of U-tube manometer should be same.

Pressure above 'A-A' in the left column = $p + \rho_1 \times g \times h_1$

Pressure above 'A-A' in the right column = $\rho_2 \times g \times h_2$.

Hence equating the above two equations.

$$p + \rho_1 \times g \times h_1 = \rho_2 \times g \times h_2.$$

$$\boxed{p = (\rho_2 \times g h_2 - \rho_1 \times g \times h_1)}$$

Now consider the datum line Y-Y as shown in fig. then pressure in the right limb above Y-Y equal to $\rho_2 g \times (\Delta h + h_2)$. Pressure in left limb above Y-Y

$p_A + (\rho_1 \times g \times (\Delta h + h_1))$

Equating the Pressures.

$$p_A = (\rho_2 g \times (\Delta h + h_2)) - (\rho_1 g \times (\Delta h + h_1)) = \rho_2 g \Delta h + \rho_2 g h_2 - \rho_1 g \Delta h - \rho_1 g h_1$$

$$= \Delta h (\rho_2 g - \rho_1 g) + \rho_2 g h_2 - \rho_1 g h_1$$

$$p_A = \frac{a \times h_2}{A} (\rho_2 g - \rho_1 g) + \rho_2 g h_2 - \rho_1 g h_1$$

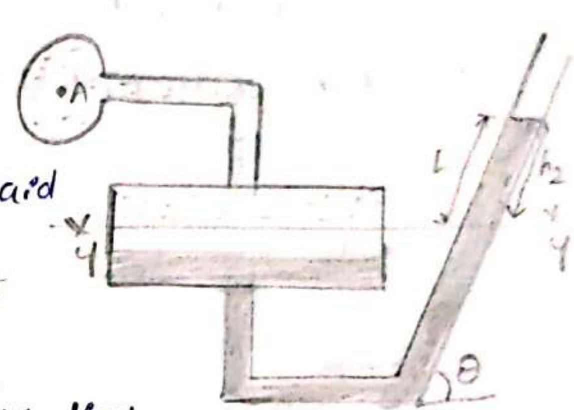
As the area 'A' is very large as compared to 'a' hence ratio a/A becomes very small & can be neglected then $p_A = h_2 (\rho_2 g - \rho_1 g) + \rho_2 g h_2 - \rho_1 g h_1$

$$\boxed{p_A = \rho_2 g h_2 - \rho_1 g h_1}$$

from the above Eqn it is clear that ρ_1, h_1 is known & hence knowing (h_2) & rise of heavy liquid in the right limb

Inclined single column manometer:-

fig. shows inclined single column manometer.



- This manometer is more sensitive due to inclination the distance moved by the heavy liquid in the right limb will be more.

l = length of the heavy liquid moved in right from x-x.

θ = Inclination of right limb with horizontal.

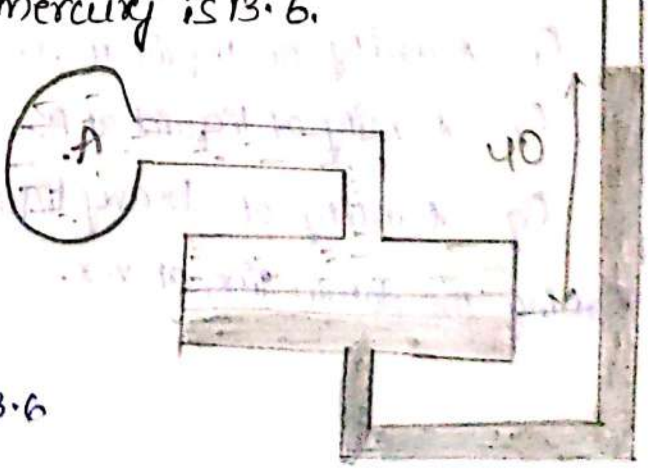
h_2 = vertical ris of heavy liquid in right limb from x-x $= L \times \sin\theta$.

The pressure at 'A' is $p_A = h_2 \rho_2 g - h_1 \rho_1 g$.

Sub. the value of $h_2 = L \times \sin\theta$

$$\boxed{p_A = L \times \sin\theta \, \rho_2 g - h_1 \rho_1 g}$$

A single column manometer is connected to a pipe containing the liquid of sp. gravity 0.9 as shown in fig. Find the pressure in the pipe. if the area of reservior is 100 time the area of the tube, for the manometer reading. The specific gravity of mercury is 13.6.



Sol/ Given,

Sp. gr of liquid = 0.9

Density of fluid $\rho_1 = 1000 \times 0.9$
$= 900 \, kg/m^3$

20

Sp. gr of mercury = 13.6

Density of mercury $= \rho_2 = 1000 \times 13.6$

$= 13600 kg/m^3$

Height of fluid in left limb $h_1 = 20cm = 0.2m$

Height of fluid in right limb $h_2 = 40cm = 0.4m$

Area of reservoir = 100 times the area of tube $= \frac{A}{a} = 100$.

$$P_A = \frac{a h_2}{A}\left[S_2 g - S_1 g\right] + \left[S_2 g h_2 - S_1 g h_1\right]$$

$$= \frac{1}{100}\times(0.4)\left[13600\times 9.81 - 900\times 9.81\right] + \left[13600\times 9.81\times 0.4 - 900\times 9.81\times 0.2\right]$$

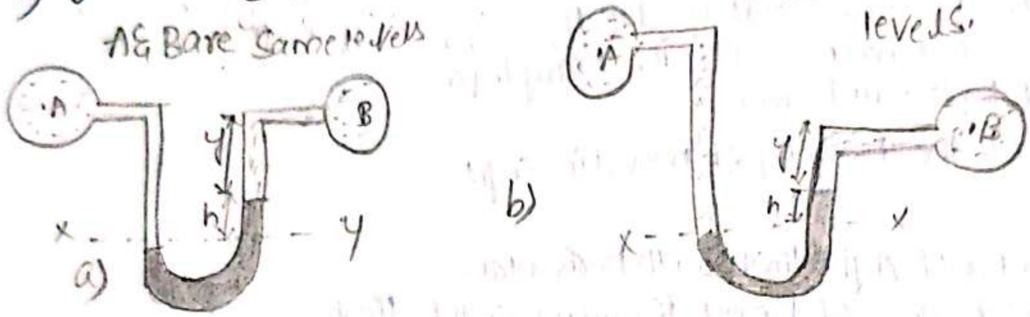
$$\boxed{P_A = 5209.894 \, N/m^2}$$

Differential manometers:-

1. U-tube manometer 2. Inverted U-tube manometer.
3. Micromanometer.

i) U-tube Differential manometer:-

A & B are same levels — A & B are different levels.



In fig. a the two points A & B are at different levels and also contains liquids of different specific gravities. These points are connected to the U-tube differential manometer. Let the pressure at A & B are

h = Difference of mercury level in the U-tube.

y = Distance of centre of 'B', from the mercury level in the right limb.

x = Distance of the centre of 'A' from the mercury level in the right limb.

ρ_1 = Density of liquid at 'A'.

ρ_2 = Density of liquid at 'B'.

ρ_g = Density of heavy liquid (or) mercury.

Taking the datum line at X-X.

Pressure about x-x in left limb $= P_A + (h+x)\rho_1 g$

Pressure about x-x in right limb $= \rho_2 g y + \rho_g g h + P_B$

Equating the two pressures

$$\rho_1 g(h+x) + P_A = \rho_g \times g \times h + \rho_2 g \times y + P_B$$

$$P_A - P_B = \rho_g \times g \times h + \rho_2 \times g \times y - \rho_1 g h - \rho_1 g x$$

$$\boxed{P_A - P_B = hg(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x}$$

In fig b. The two points A & B are at the same levels and contains the same liquid of density ρ_1 then pressure about x-x in right limb
$= \rho_g g h + \rho_1 g x + P_B$

The pressure at x-x line in left limb $= \rho_1 g(h+x) + P_A$.

Equating the two equations (or) pressures

right limb = left limb.

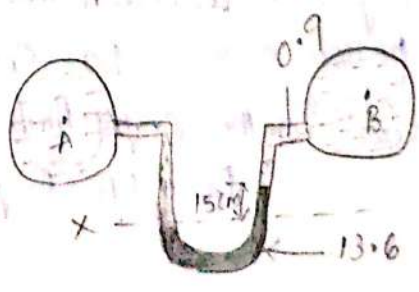
$$\rho_g g h + \rho_1 g x + P_B = \rho_1 g(h+x) + P_A$$

$$P_A - P_B = \rho_g g h + \rho_1 g x - \rho_1 g h - \rho_1 g x$$

$$\boxed{P_A - P_B = gh(\rho_g - \rho_1)}$$

A pipe contains an oil of sp.gr 0.9. A differential manometers connected at two points A & B shows pressure difference at mercury level as 15cm. Find the difference of pressure at two points.

Sol: Given,



sp.gr of an oil $= 0.9$

Density of an oil $\rho_1 = 1000 \times 0.9$
$= 900\,kg/m^3$.

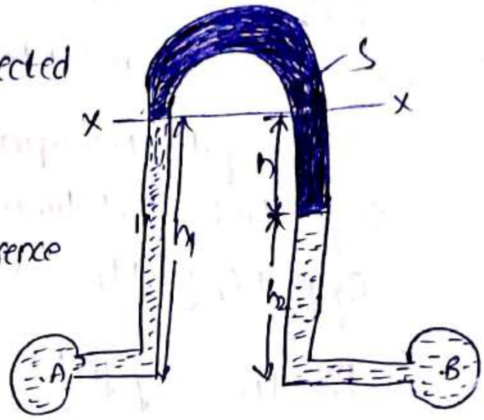
Sp. gravity of mercury = 13.6.
Density of mercury ρ_g = 1000×13.6
$= 13600\ kg/m^3$.
$\therefore h = 15\ cm = 0.15\ m$.

$$P_A - P_B = gh[\rho_g - \rho_1] = 9.81 \times 0.15[13600 - 900]$$
$$P_A - P_B = 18688.05\ N/m^2$$

Date :- 6-1-2021

Inverted U-tube differential manometer:-

It consist of an inverted U-tube containing a light liquid.
The two ends of the tube are connected to the points where difference of pressure is to be measured.
It is used for measuring difference of low pressure as shown in fig. Connected to the two points A & B



let, the pressure at 'A' is more than the pressure at 'B'.

h_1 = height of the liquid in the left limb. bellow the datum line.
h_2 = height of the liquid in right limb.
h = Difference of light liquid.
ρ_1 = Density of liquid at 'A'.
ρ_2 = Density of liquid at 'B'.
ρ_g = Density of light liquid.
P_A = pressure at 'A'.
P_B = Pressure at 'B'.

Taking X-X as datum line then the pressure in the left limb bellow X-X $= P_A - \rho_1 g h_1$ —— (1).

Pressure in the right limb below X-X

$$P_B = P_B - \rho_2 g h_2 - \rho_g g h \text{ —— } (2).$$

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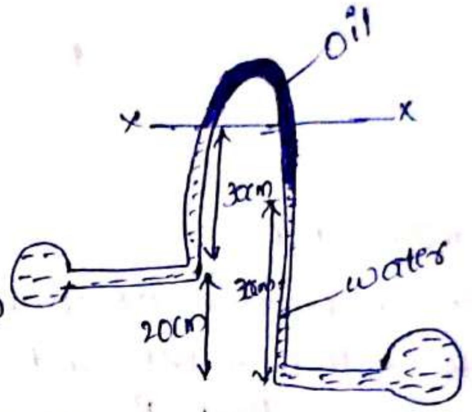
Equating the (1) & (2).

$P_A - \rho_1 g h_1 = P_B - \rho_3 g h_3 - \rho_g g h$.

$P_A - P_B = \rho_1 g h_1 - \rho_3 g h_3 - \rho_g g h$.

$$\boxed{\therefore P_A - P_B = \rho_1 g h_1 - \rho_3 g h_3 - \rho_g g h.}$$

1) In figure an inverted differential manometer is connected to two pipes A&B which convey water-the fluid in manometer is oil of sp.gr 0.8 for manometer reading shown in fig. find the pressure difference between A&B.



Sol: Given,

sp. gravity of oil $= 0.8 N/m^2$

Density of oil $\rho_2 = 1000 \times 0.8$

$= 800 \, kg/m^3$

Difference of oil two limbs $= (30+20)-30$

$= 20$.

Taking datum line at x-x

Pressure at left limb below $x-x = P_A - \rho_1 g h_1$

$= P_A - 1000 \times 9.81 \times 0.3$

$= P_A - 2943$ — (1)

pressure at right limb $x-x = P_B - \rho_2 g h_2 - \rho_g g h$

$= P_B - 800 \times 9.81 \, 0.3 - 800 \times 9.81 \times 0.2$

$= P_B - 4512.6$ — (2).

Equating (1) & (2) $\Rightarrow$ $P_A - 2943 = P_B - 4512.6$.

$P_A - P_B = -4512.6 + 2943$.

$$\boxed{P_A - P_B = -1569.6 \, N/m^2}$$

Find out the differential reading 'h' of an inverted U-tube manometer containing oil of sp.gra. 0.7 as the manometric fluid, when connected to pipes A&B as shown in figure below conveying liquid of sp.gr. 1.2 & 1.0 and immiscible with manometric fluid pipes A&B are located at same level assume the pressure at A&B are same.

Sol: Given,

Specific gravity of oil $= 0.7$

Density of

Micromanometer:-

Micromanometers are used to determine very small pressure difference between two points.

Part - B

Hydrostatic forces on fluids surfaces:-

It deals with the fluids that is liquids, gaseous at rest This means there will be no relative motion between adjacent (or) naibouring fluid. The velocity gradient which is equal to charge of velocity which is equal to change of velocity between two adjacent layers of fluid is divided by the distance between the layers will be zero (or) $\frac{du}{dy} = 0$. The shear stress which is equal to $\tau = \mu\left(\frac{du}{dy}\right)$ will be zero. Then the forces acting on the fluid particals will be (i) Due to pressure of fluid.

ii) Due to gravity (or) self weight of fluid particle.

Total pressure and centre of pressure:-

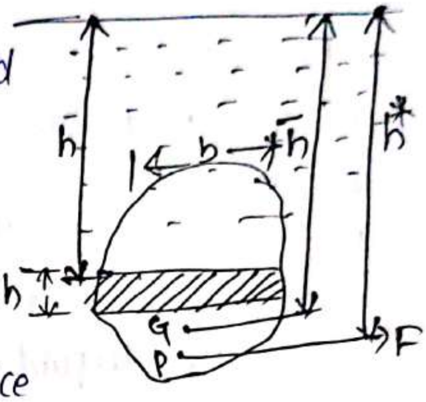
Total pressure is defined as the force excited by a static fluid on the surface either plane or curved, when the fluid comes in contact with the surfaces. This force always acts normal to this surface.

centre of pressure is defined as point of application of the total pressure on the surface. There are 4 case of submerged surfaces on which the total pressure force and centre of pressure is to be determined as follows. The submerged surfaces may be

i. vertical plane surface
ii. Horizontal plane surface
iii. Inclined plane surface
iv. curved plane surface.

Vertical plane surface:-

consider a vertical plane surface of an arbitary shape submerged in a liquid as shown in figure.



let,

A = Total area of the surface.

$\bar{h}$ = Distance of centre of gravity of the area from the free surface of liquid

G = centre of gravity of the plane surface

P = centre of pressure

h^* = Distance of centre of pressure from free surface of liquid.

Total Pressure (F):-

The total pressure on the surface may be determined by dividing the entier surface into a no. of small parallel strips.. The force on a small strip is the calculated and the total pressure force on the whole area is calculated by intipretis the force on small strip.

consider a strip of thickness 'dh' and width 'b' and depth 'h' from free surface of liquid as shown in fig.

Pressure intensity on the strip $P = \rho g h$

Area of the strip $A = b \times dh$

Total pressure force on strip $dF = P \times A$

$$= \rho g \times h \times b \times dh$$

$\therefore$ Total pressure force on the whole surface

$$F = \int dF$$

$$= \int \rho g h \times b dh$$

$= \rho g \int h \times b \times dh$

$= \rho g \int h\, dA$.

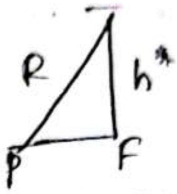
= moment of surface area above the free surface of liquid.

Area of surface × distance of centre of gravity from free surface

$$\boxed{F = \rho g A \bar{h}.}$$

Centre of pressure (h^*):

Centre of pressure is calculated by using the "principle of manometers". Such states that moment of resultant force about an axis is equal to the sum of moments of the components about the same axis. The resultant force 'F' is acting at 'P' at a distance h^*. From the free surface of the liquid as shown in fig.



eg: R = P+Q

Hence, moment of the force F above free surface of the liquid $= F \times h^*$ —— (1)

moment of the force dF acting on a strip above free surface of the liquid

$= dF \times h$

$= \rho \times g \times h \times b \times dh$.

Sum of moments of all such forces about free surfaces at liquid $= \int \rho g h \times b \times dh \times h$

$= \rho g \int b \times dh \times h^2$

$= \rho g \int dA \times h^2$

but $\int h^2 dA = \int h^2 \times b \times dh$

= moment of inertia of the surface about free surface of liquid $= I_0$

Sum of the moments about free Surface = $\rho g I_0$ —— (2)

Equating (1) & (2).

$$F \times h^* = \rho g I_0$$

$$\rho g A \times \bar{h} \times h^* = \rho g I_0$$

$$\boxed{h^* = \frac{I_0}{A \times \bar{h}}} \quad \text{—— (3).}$$

By the theorem of parallel axis.

$$I_0 = I_G + A\bar{h}^2.$$

$$h^* = \frac{I_G + A\bar{h}^2}{A \times \bar{h}}$$

$$h^* = \frac{I_G}{A\bar{h}} + \frac{A\bar{h}^2}{A\bar{h}}$$

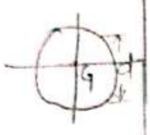
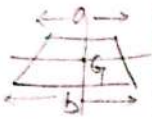
$$\boxed{h^* = \frac{I_G}{A\bar{h}} + \bar{h}} \quad \text{—— (4).}$$

Here; $\bar{h}$ is the distance of centre of gravity of the area of vertical Surface from free Surface of liquid. Hence equation (4) it is clear that.

i) center of pressure (h^*) lies bellow the centre of gravity of the vertical Surface.

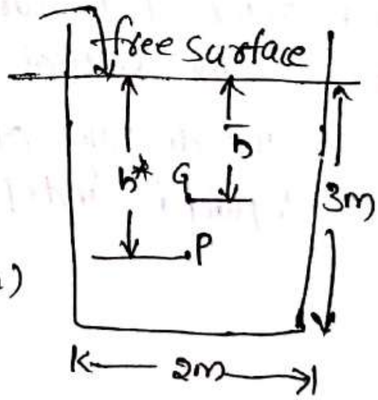
ii) The distance of centre of pressure from the Surface of liquid is independent of the density of liquid.

Geometric properties of plane surface.

Plane Surface	C.G from base	Area	moment of inertia about an axis from through CG	moment of inertia about base (I_0).
	$x = d/2$	$b \times d$	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
	x	$\frac{1}{2}bh$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$
	$x = d/2$	$\frac{\pi}{4} \times d^2$	$\frac{\pi d^4}{64}$	—
	$x = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$	$\left(\frac{a+b}{2}\right)h$	$\left(\frac{a^2+4ab+b^2}{36 \times (a+b)}\right)h^3$	—

A rectangular plane surface is 2M width & 3m deep it lies in vertical plane in water determine the total Pressure & Position of centre of pressure on the plane surface when it's upper edge is horizontal & a) co-inside with water surface.
b) 2.5m bellow the free water surface.

Sol:- Given,
width $b = 2m$
depth $d = 3m$



a) Upper edge co-insides with water surface
Total pressure $F = \rho g \bar{h} A$.

$$= 1000 \times 9.81 \times \frac{3}{2} \times (3 \times 2)$$
$$F = 88290 N/m^2.$$

centre of Pressure $h^* = \frac{I_G}{A\bar{h}} + \bar{h}$

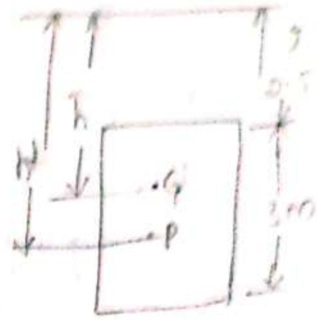
$$I_G = \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5 m^4$$
$$h^* = \frac{4.5}{6 \times 1.5} + 1.5 = 2m.$$

b) upper edge is 2.5 m bellow water surface.

Total pressure $F = \rho \times g \times \bar{h} \times A$

$$= 1000 \times 9.81 \times \left(\tfrac{3}{2} + 2.5\right) \times (3 \times 2)$$

$$F = 235440 \ N/m^2$$



Centre of pressure $h^* = \dfrac{I_G}{A\bar{h}} + \bar{h}$

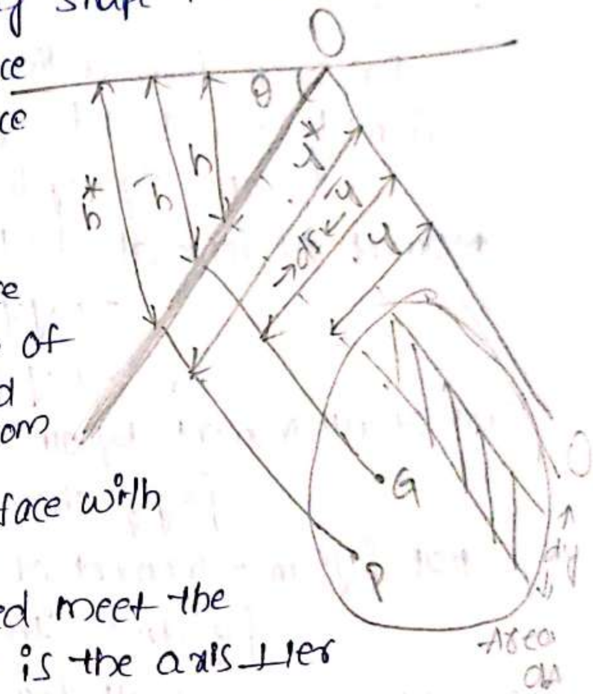
$$= \frac{4.5}{6 \times 4} + 4$$

$$\therefore h^* = 4.187 \ m.$$

Date :- 9-1-2021.

Inclined plane Surface :-

consider a plane Surface of orbitary shape immersed in a liquid in Such a way the plane of the surface makes an angle 'θ' with the free Surface of the liquid as shown in fig.



let,

A = Total area of the inclined surface

h^* = Depth of C.P from free surface of liquid

$\bar{h}$ = Depth of C.G of inclined area from the surface of water.

θ = Angle made by the plane of Surface with the free body water surface.

let the plane of Surface it produced meet the free liquid Surface at 'O' Then O-O is the axis ⊥ler to the plane of the surface.

$\bar{y}$ = Distance of C.G of the inclined surface from the axis 'O-O'

y^* = Distance of C.P from 'O-O'.

consider a small strip of area 'dA' at a depth 'h' from free surface and a distance of 'y' from 'O-O'.

pressure intensity on the strip $P = \rho g h$

∴ pressure force 'dF' on the strip = $P \times A$.

$$dF = \rho g h \times dA.$$

Total pressure force on the whole area $\int dF = \int \rho g h \times dA$

$$F = \rho g \int h \, dA$$

$$\boxed{F = \rho g A \bar{h}}$$

But from fig: $\frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*} = \sin\theta$

then $\frac{h}{y} = \sin\theta$

$h = y\sin\theta$

$\int dF = \int \rho g h \times dA$

$F = \rho g \int h\,dA$

$= \rho g \int y\sin\theta\, dA$

$= \rho g \sin\theta \int y\,dA$

$F = \rho g \sin\theta\, \bar{y} A = \rho g \sin\theta \frac{\bar{h}}{\sin\theta} \cdot A$

$$\boxed{\therefore F = \rho g \bar{h} A}$$

C.P h^* :

Pressure force on the strip $dF = \rho g h\, dA$

from $\frac{h}{y} = \sin\theta \Rightarrow h = y\sin\theta$

$dF = \rho g \times y\sin\theta \times dA$

Moment of force dF about axis '0-0' $= dF \times y$

$= \rho g y \sin\theta\, dA \times y$

$= \rho g y^2 \sin\theta\, dA$

Sum of all moments by all such forces about '0-0' $= \int \rho g y^2 \sin\theta\, dA$

$= \int \rho g y^2 \sin\theta\, dA = \rho g \sin\theta \int y^2 dA$.

But $\int y^2 dA$ = moment of inertia of the surface that '0-0'.

$\int y^2 dA = I_0$

Sum of moment of all forces about '0-0' $= \rho g \sin\theta\, I_0$ —— (1)

moment of total force 'F' about '0-0' axis is given by $= F \times y^*$ —— (2)

Now equating (1) & (2)

$F \times y^* = \rho g \sin\theta \cdot I_0$

$y^* = \frac{\rho g \sin\theta I_0}{F}$ —— (3)

$\frac{h^*}{\sin\theta} = \frac{\rho g \sin\theta \cdot I_0}{F}$

$I_0 = I_G + A\bar{y}^2$

Substitute 'I_0' in eqn (3)

$\frac{h^*}{\sin\theta} = \frac{\rho g \sin\theta (I_G + A\bar{y}^2)}{F}$

$$\frac{h^*}{\sin\theta} = \frac{\rho g \sin\theta (I_G + A\bar{y}^2)}{\rho g A\bar{h}}$$

$$\frac{h^*}{\sin\theta} = \frac{\rho g \sin\theta}{\rho g A\bar{h}} [I_G + A\bar{y}^2]$$

$$\boxed{h^* = \frac{\sin^2\theta}{A\bar{h}} [I_G + A\bar{y}^2].}$$

$$h^* = \frac{\sin^2\theta}{A\times\bar{y}\sin\theta} [I_G + A\bar{y}^2]$$

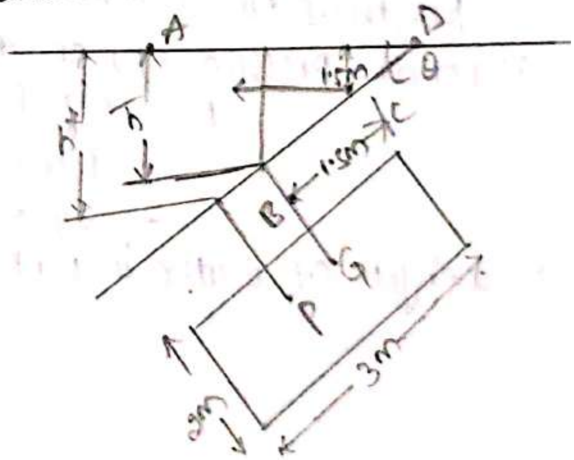
$$h^* = \frac{\sin\theta}{A\bar{y}} [I_G + A\bar{y}^2].$$

$$h^* = \frac{\sin\theta}{A\cdot\frac{\bar{h}}{\sin\theta}} \left[I_G + A\left(\frac{\bar{h}}{\sin\theta}\right)^2\right]$$

$$h^* = \frac{\sin^2\theta}{A\times\bar{h}} [I_G + A].$$

$$\boxed{h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}.}$$

(1) A rectangular plane surface 2m wide & 3m deep lies in water in such a way that it's plane makes an angle of 30° with the free surface of water. Determine the total pressure and position of C.P when the upper edge is 1.5m bellow the free water surface



Sol: Given,
width $b = 2m$ and Depth $d = 3m$

$\theta = 30°$.

Distance from upper edge $= 1.5m$

Area of plane $A = 3\times2 = 6m^2$

Total pressure $F = \rho g \bar{h} A$

$\bar{h} = 1.5 + 1.5\sin 30° = 2.25m$

$F = 1000\times 9.81 \times 2.25 \times 6.$

$\boxed{F = 132435\ N.}$

centre of pressure $h^* = \dfrac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$

$$I_G = \frac{bd^3}{12} = \frac{2\times3^3}{12} = 4.5.$$

$$h^* = \frac{4.5 \times \sin^2(30°)}{6\times 2.25} + 2.25$$

$\boxed{h^* = 2.333m.}$

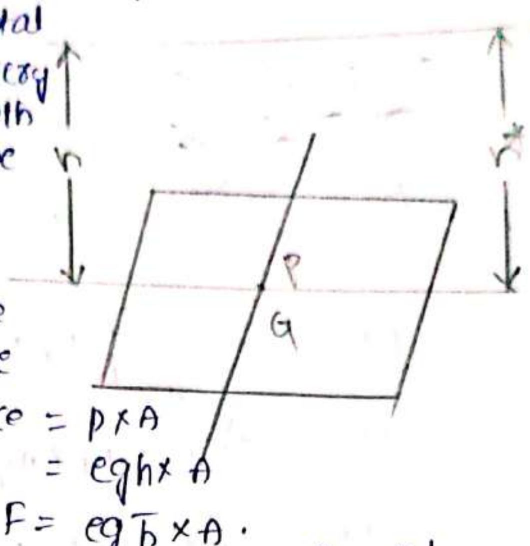
Horizontal plane surface submerged in liquid:

Consider a plane surface horizontal surface immersed in a static fluid as every point of the surface is at same depth from the free surface of the liquid the pressure intensity will be equal to the centre surface and equal to $p = \rho g h$

where 'h' is the depth of surface

let A = Total area of surface

Then total force 'F' on the surface $= p \times A$



$$= \rho g h \times A$$

$$F = \rho g \bar{h} \times A.$$

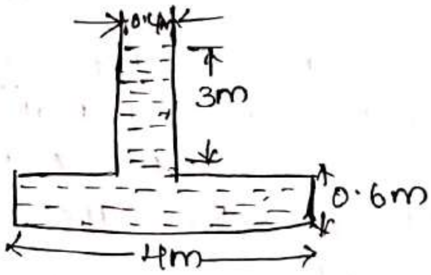
where $\bar{h}$ = Depth of C.G from free surface of liquid

$$\bar{h} = h.$$

$$h^* = h.$$

① Fig shows a tank of water find a) Total pressure on the bottom of the tank b) weight of water in tank c) Hydrostatic paradox between the results of a & b width of tank is 2m.

Sol: Given,



Depth of water of an bottom of tank

$h_1 = 3 + 0.6 = 3.6m$

width of tank = 2m

length of tank bottom $A = 4 \times 2 = 8m^2$

i) Total pressure (F) on the bottom is

$$F = \rho g A h$$

$$= 1000 \times 9.81 \times 8 \times 3.6.$$

$$= 282528N$$

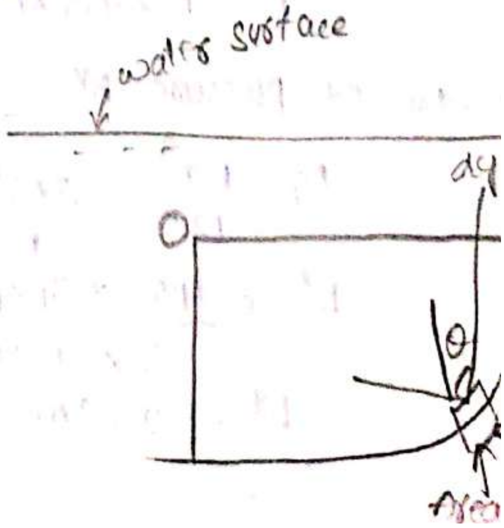
ii) weight of water in tank $= \rho g \times$ volume of tank

$$= 1000 \times 9.81 \times [3 \times 0.4 \times 2 + 4 \times 0.6 \times 2]$$

$$= 1000 \times 9.81 [2.4 + 4.8]$$

$$= 70632 N.$$

Curved surface submerged in liquid:



consider a curved surface 'AB' submerged in static fluid fluid. let 'dA' is the area of a small strip at a depth of 'h' from free water surface.

when pressure intensity on dA is $P = \rho g h$

on pressure force $dF = P \times A$

$$dF = \rho g h \times dA.$$

This force dF acts normal to the surface. Hence total pressure force on the curved face should be $F = \int \rho g h \, dA$

but here as the direction of forces on the surface area are not in the same direction but varies from point to point

By resolving the force dF into two components i.e. dF_x & dF_y. The total force F_x and F_y are obtained from intigra -ting dF_x & dF_y.

$\therefore$ The total force on the curved surface $F = \sqrt{F_x^2 + F_y^2}$.

Inclination of resultant with horizontal is '$\tan\theta$' $= \dfrac{F_x}{F_y}$

$$\boxed{F_y = \rho g \int h \, dA \cos\theta}$$

<u>Buoyancy</u>:-

when a body is immersed in a fluid an upward force is exerted by the fluid on the body. This upward force is equal to the weight of fluid displaced by the body and the force is called "Buoyancy".

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UNIT-3.

Fluid Kinematics

Kinematics of fluid motion:-

Study of fluid particals which are motion without considering forces is called as "fluid kinematics." There are two types they are.

1. Lagrangian method. 2. Euler method.

In lagrangian method a single fluid partical is followed during it's motion and its velocity, acceleration, density.. etc. are described.

In Euler method the velocity, acceleration, pressure density -- etc are described at a point in flow fluid. The euler method is commonly used in fluid mechanics.

Classification of flows:-

1. Steady and un-steady flow:-

Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density etc. donot change with respect to time. $\left(\frac{\partial v}{\partial t}\right)_{(x_0,y_0,z_0)} = 0 ; \left(\frac{\partial P}{\partial t}\right)_{(x_0,y_0,z_0)} = 0 ; \left(\frac{\partial \rho}{\partial t}\right)_{(x_0,y_0,z_0)} = 0$.

Un-steady flow is that type of flow in which the velocity, pressure & density at a point changes with respect to time.

$$\left(\frac{\partial v}{\partial t}\right)_{(x_0,y_0,z_0)} \neq 0 ; \left(\frac{\partial P}{\partial t}\right)_{(x_0 y_0 z_0)} \neq 0 ; \left(\frac{\partial \rho}{\partial t}\right)_{(x_0,y_0,z_0)} \neq 0.$$

2. Uniform & Non-uniform flow:-

Uniform flow is defined as that type of flow in which velocity at any given time does not change with respect to space (or) displacement.

$\left(\frac{\partial V}{\partial s}\right) = 0$ where; ∂V = Change of velocity ∂S = length of flow in direction

Non-uniform flow is that type of flow in which the velocity at any given times changes with respect to space $\left(\frac{\partial V}{\partial s}\right) \neq 0$.

3. laminar & Turbulent flow:-

laminar flow is defined as that type of flow in which fluid particals move along well defined path (or) stream line and all the stream lines are straight and parallel.

Turbulent flow is that type of flow in which fluid particals move in a zig-zag way. Due to moment of fluid particals in zig zag way the eddies formation takes place which are responsible for high energy loss. for a pipe flow, the type of flow is determined by a non-dimensional number $\left(\frac{\rho V D}{\mu}\right)$ called reynolds number = $\frac{\text{Inertia force}}{\text{Viscos force}}$.

$R_e > 4000$ = Turbulent flow.

$R_L < 2000$ = laminar flow.

4. Compressible & In-compressible flows:-

compressible flow is that type of flow in which density of fluid changes from point to point (or) the density is not constant for the fluid thus $\rho \neq$ constant. Air

In compressible flow is that type of flow in which the density is constant for the fluid flow. liquids are generally in compressible while gases are compressible. water

Thus ρ = constant.

5. Rotational & Irrotational flow;

Rotational flow is that type of flow in which fluid Particals while flowing along Streamline also rotate about their own axis and if the fluid particals donot rotate about their own axis and that type of flow is knownas "irrotational flow".

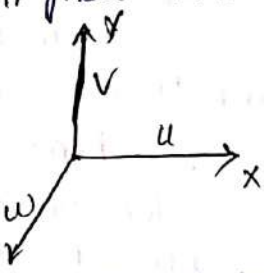
6. One, two and three dimensional flows:

One-dimensional flow is that type of flow in which the flow parameters such as velocity is a function of time and one phase co-ordinates only say x. For steady one dimensional flow the velocity is a faction of one-space co-ordinate only the variation of velocity in other two mutually ⊥ler directions is assumed neglissible.

$u = f(x)$; $v = 0$; $w = 0$.

Two-dimensional flow is that type of flow in which velocity is a function of time and two rectangular space co-ordinates say x & y the variation of velocity in third direction is negligible. Thus

$u = f_1(x,y)$; $v = f_2(x,y)$; $w = 0$.



Third-dimensional flow it is the type of flow in which velocity is a function of time & three mutually ⊥ler directions. But for a study three dimensional flow the fluid perameters are the function of three co-ordinates x, y, z only.

Thus $u = f_1(x,y)$; $v = f_2(x,y)$; $w = f_3(x,y)$.

Rate of flow (or) Discharge (Q):

7. Real and Ideal fluids.

Date :- 8-2-2021

The diameter of pipe at sections (1) & (2) are 10cm & 15cm respectively. Find discharge through the pipe if velocity of water at section (1) is 5 m/sec.

Determine also the velocity at section (2).

Sol: Given,

Diameter of pipe at Section (1) = 10cm = 0.1m.

Diameter of pipe at Section (2) = 15cm = 1.5m.

velocity (v_1) = 5 m/sec.

velocity (v_2) = ?

Area of pipe at (1) $= \frac{\pi}{4} \times d_1^2$

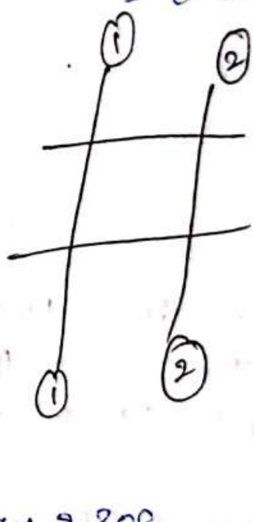
$= \frac{\pi}{4} \times (0.1)^2$

$= 7.85 \times 10^{-3} m^2$

Area of pipe (2) $= \frac{\pi}{4} \times d_2^2$

$= \frac{\pi}{2} (1.5)^2$

$= 0.017 m^2$.



By using continuity eqn.

$A_1 V_1 = A_2 V_2$

$5 \times 7.85 \times 10^{-3} = 0.017 \times V_2$.

$\frac{7.85 \times 10^{-3} \times 5}{0.017} = V_2$

$\boxed{\therefore V_2 = 2.308 \text{ m/sec}}$

Discharge $Q = A_1 V_1 = A_2 V_2$

$= 7.85 \times 10^{-3} \times 5$

$= 0.039 \text{ m}^3/\text{sec}$

(or) $Q = A_2 V_2$

$= 0.017 \times 2.308$

$= 0.039 \text{ m}^3/\text{sec}$.

(2) A 30cm dia pipe conveying water branches into 2 pipes of diameter 20cm & 15cm respectively. If the average velocity in 30cm dia of pipe is 2.5 m/sec. Find the discharge in this pipe. Also determine the velocity in 15cm pipe. If the average velocity 20cm pipe is 2m/sec.

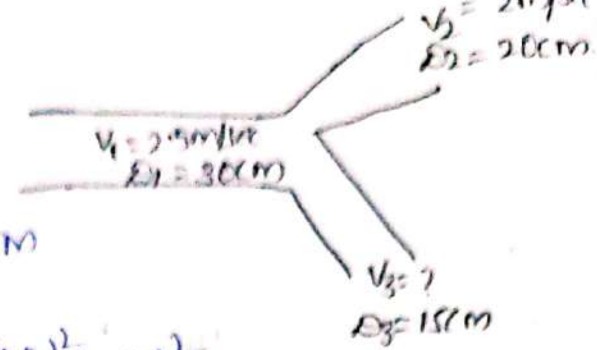
Sol: Given,

Diameter of pipe Section (1) = 30cm = 0.3m

velocity of pipe (1) = 2.5 m/sec.

Area of pipe $A_1 = \frac{\pi}{4} \times d^2 = 0.0706 m^2$

Diameter (d_2) = 20cm = 0.2 m
velocity (v_2) = 2m/sec.
Area (A_2) = $\frac{\pi}{4} \times (20\times10^{-2})^2$
= $0.03141 m^2$
Diameter (d_3) = 15 cm = 0.15 m = 0.15M
velocity (v_3) = ?
Area of pipe (A_3) = $\frac{\pi}{4} \times d^2 = \frac{\pi}{4} (0.15)^2$
= $0.0176\ m^2$.

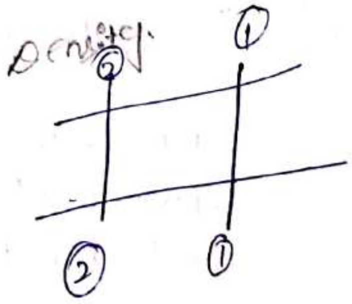


$Q_1 = A_1 V_1 = 0.0706 \times 2.5 = 0.1765\ m^3/sec$
$Q_2 = A_2 V_2 = 0.03141 \times 2 = 0.0628\ m^3/sec$.
$Q_3 = A_3 V_3 = ?$
$Q_1 = Q_2 + Q_3$
$Q_3 = Q_1 - Q_2$
$= 0.1765 - 0.0628$
$Q_3 = 0.1136\ m^3/sec$
$Q_3 = A_3 V_3$
$\frac{Q_3}{A_3} = V_3 \Rightarrow \frac{0.1136}{0.0176} = V_3$

$\boxed{\therefore V_3 = 6.45 m/sec.}$

(3) A 25cm dia of pipe carries oil of sp.gr. 0.9 at a velocity of 3m/sec at another section the dia 20cm find the velocity at this section & Also mass rate of flow of oil.

Sol: Given,



Diameter of pipe = 25cm = 0.25m
$A_1 = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (0.25)^2 = 0.0490 m^2$
$V_1 = 3m/sec$
$D_2 = 20cm = 0.2m$
$A_2 = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (0.2)^2 = 0.0314 m^2$.
$V_2 = ?$

$A_1 V_1 = A_2 V_2$
$0.049 \times 3 = 0.0314 \times V_2$
$\frac{0.049 \times 3}{0.0314} = V_2$ $\boxed{\therefore V_2 = 4.681\ m/sec}$

Rate of flow of oil = $\rho_1 \times A_1 \times V_1$
$= 900 \times 0.0490 \times 3$
$= 132.3\ m^3/sec$.

Rate of flow or Discharge (Q):

It is defined as the quantity of fluid flowing per second through a section of pipe or a channel for an incompressible fluid (or liquid) the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second. For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section.

i) For liquids the units of Q are m^3/s or litres/sec.

ii) For gases the units of 'Q' is kgf/s or Newtons/s.

Consider a liquid flowing through a pipe in

A = Cross-sectional area of pipe.

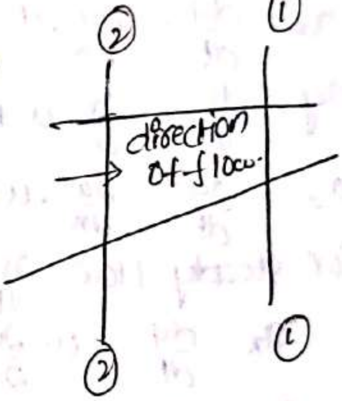
v = Average velocity of fluid across the section.

$$\boxed{Q = AV}$$

Continuity Equation:-

The equation based on the principle of conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant.

Consider '2' c/s of a pipe as shown in fig.



V_1 = Average velocity at cross-section ①-①

ρ_1 = Density at section ①-①

A_1 = Area of pipe at section ①-①

V_2, ρ_2, A_2 are corresponding values at section ②-②

Then the rate of flow at section ①-① = $\rho_1 A_1 V_1$

The rate of flow at section ②-② = $\rho_2 A_2 V_2$

According to law of conservation of mass

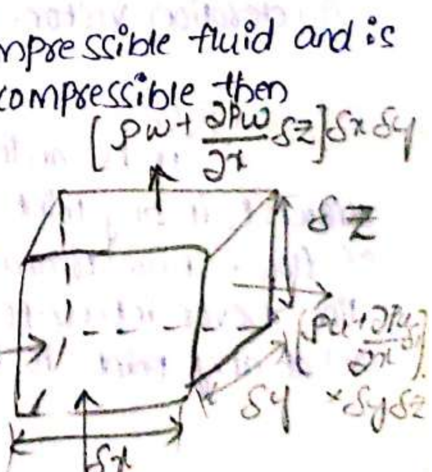
Rate of flow at section ①-① = Rate of flow at section ②-②

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad \text{—} ①$$

Eqn ① is applicable in compressible as well as incompressible fluid and is called 'continuity equation' when the fluid is incompressible then eqn reduces to $\boxed{A_1 V_1 = A_2 V_2}$

[Related problems are in front papers]

continuity in three directions



$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0}$$

Velocity and acceleration:-

let 'V' is the resultant velocity at any point in the fluid flow, let u, v, w its components in x, y, z directions. The velocity components are function of space co-ordinates and time. mathematically the velocity components are

$$u = f_1(x, y, z, t)$$
$$v = f_2(x, y, z, t)$$
$$w = f_3(x, y, z, t).$$

$\therefore$ Resultant velocity $V = u\hat{i} + v\hat{j} + w\hat{k}$

$$= \sqrt{u^2 + v^2 + w^2}$$

let a_x, a_y, a_z are the total acceleration in x, y, z directions respectively. Then by the chain rule of differentiation we have;

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt} + \frac{\partial u}{\partial t}.$$

$$a_y = \frac{dv}{dt} = \frac{\partial v}{\partial x}\frac{dx}{dt} + \frac{\partial v}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial v}{\partial z}\frac{dz}{dt} + \frac{\partial v}{\partial t}.$$

$$a_z = \frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt} + \frac{\partial w}{dt}.$$

$$u = \frac{dx}{dt}, \quad v = \frac{dy}{dt}, \quad w = \frac{dw}{dt}.$$

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\cdot u + \frac{\partial u}{\partial y}\cdot v + \frac{\partial w}{\partial z}\cdot w + \frac{\partial u}{\partial t}.$$

$$a_y = \frac{dv}{dt} = \frac{\partial v}{\partial x}\cdot u + \frac{\partial v}{\partial y}\cdot v + \frac{\partial w}{\partial z}\cdot w + \frac{\partial v}{\partial t}.$$

$$a_z = \frac{dw}{dt} = \frac{\partial w}{\partial x}\cdot u + \frac{\partial w}{\partial y}\cdot v + \frac{\partial w}{\partial z}\cdot w + \frac{\partial w}{\partial t}$$

for steady flow $\frac{\partial v}{\partial t} = 0$ was well as $\frac{\partial u}{\partial t} = 0$ & $\frac{\partial w}{\partial t} = 0$

$$a_x = \frac{du}{dt} = u\cdot\frac{\partial u}{\partial x} + v\cdot\frac{\partial u}{\partial y} + w\cdot\frac{\partial u}{\partial z}$$

$$a_y = \frac{dv}{dt} = u\cdot\frac{\partial v}{\partial x} + v\cdot\frac{\partial v}{\partial y} + w\cdot\frac{\partial v}{\partial z}.$$

$$a_z = \frac{dw}{dt} = u\cdot\frac{\partial w}{\partial x} + v\cdot\frac{\partial w}{\partial y} + w\cdot\frac{\partial w}{\partial z}.$$

Acceleration vector $= a_x\hat{i} + a_y\hat{j} + a_z\hat{k} = \sqrt{a_x^2 + a_y^2 + a_z^2}$.

Stream line:-

It is an imaginary line (or) curve drawn in space such that a tangent drawn to it any point is velocity vector, There fore stream line gives the direction of flow. Two stream lines can never intersect it each other (or) a single stream line never intersects with it self because velocity at any given instant (or) at any point is unique.

path line:-

It is a path traced by a single fluid partical at different instants of time.

A path line can intersect with it self.

Streak line:-

It is the local of different fluid particals passing through the fixed point.

Stream tube:-

=

Local acceleration and convective acceleration:-

local attraction is defined as the rate of increase of velocity with respect to time at a given point in a flow field.

$$\left.\begin{aligned} a_x = \frac{du}{dt} &= \frac{\partial u}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial u}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt} + \frac{\partial u}{\partial t}. \\ a_y = \frac{dv}{dt} &= \frac{\partial v}{\partial x}\cdot\frac{dv}{dt} + \frac{\partial v}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial v}{\partial z}\cdot\frac{dz}{dt} + \frac{\partial v}{\partial t}. \\ a_z = \frac{dw}{dt} &= \frac{\partial w}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial w}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial w}{\partial z}\cdot\frac{dz}{dt} + \frac{\partial w}{\partial t}. \end{aligned}\right\} \text{ — } ①$$

$\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial w}{\partial t}$ are known as local attraction.

convective acceleration is defined as the rate of change of velocity due to the change of position of fluid particals in a fluid flow.

The above expression other than $\frac{\partial v}{\partial t}, \frac{\partial u}{\partial t}, \frac{\partial w}{\partial t}$ in eqn ① are known as "convective acceleration."

(4) The velocity vector in fluid is $V = 4x^3 i - 10xy j + 2t\, k$. Find the velocity and acceleration at (2,1,3) at time 1sec.

Sol:- Given,

velocity components are u, v, w.

$u = 4x^3, \quad v = -10xy, \quad w = 2t.$

$(2,1,3) \Rightarrow x = 2, \; y = 1, \; z = 3.$

$u = 4(2)^3 = 32;\ v = -10(2)(1) = -20;\ w = 2\times1 = 2.$

velocity $= ui + vj + wk = 32i - 20j + 2k.$

Resultant $= \sqrt{u^2+v^2+w^2} = \sqrt{(32)^2+(20)^2+(2)^2} = 37.78$ units.

Acceleration: $a_x i + a_y j + a_z k.$

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial u}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial u}{\partial z}\cdot\frac{dz}{dt} + \frac{\partial u}{\partial t} \quad —①$$

$$a_y = \frac{dv}{dt} = \frac{\partial v}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial v}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial v}{\partial z}\cdot\frac{dz}{dt} + \frac{\partial v}{\partial t} \quad —②$$

$$a_z = \frac{dw}{dt} = \frac{\partial w}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial w}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial w}{\partial z}\cdot\frac{dz}{dt} + \frac{\partial w}{\partial t} \quad —③$$

from ①

$$a_x = \frac{\partial}{\partial x}(4x^3)\cdot\frac{dx}{dt} + \frac{\partial}{\partial y}(4x^3)\cdot\frac{dy}{dt} + \frac{\partial}{\partial z}(4x^3)\cdot\frac{dz}{dt} + \frac{d}{dt}(4x^3).$$

$$a_x = \cancel{8}\,12x^2\cdot\frac{dx}{dt} + 0 + 0 + 0.$$

$$a_x = \frac{dx}{dt} = u \Rightarrow 12x^2\times4 = 12x^2\times4x^3 = 48x^5$$

$$= 1536.$$

from ②

$$a_y = \frac{\partial}{\partial x}(-10xy)\frac{dx}{dt} + \frac{\partial}{\partial y}(-10xy)\frac{dy}{dt} + \frac{\partial}{\partial z}(-10xy)\frac{dz}{dt} + \frac{\partial}{\partial t}(-10xy).$$

$$= -10y\frac{dx}{dt} - 10x\frac{dy}{dt} + 0 + 0.$$

$$\frac{dx}{dt} = u;\ \frac{dy}{dt} = v.$$

$$= -10y\cdot u - 10x\cdot v.$$

$$= -10y\cdot 4x^3 - 10x\cdot(-10xy)$$

$$= -40x^3y + 100x^2y = -40(2)^3 + 100(2)^2\times1 = 80.$$

from ③

$$a_z = \frac{\partial}{\partial x}(2t)\frac{dx}{dt} + \frac{\partial}{\partial y}(2t)\frac{dy}{dt} + \frac{\partial}{\partial z}(2t)\frac{dz}{dt} + \frac{\partial}{\partial t}(2t)$$

$$= 0 + 0 + 0 + 2.$$

$$a_z = 2.$$

Acceleration vector $= a_x i + a_y j + a_z k = 1536i + 80j + 2k.$

Resultant vector $= \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{(1536)^2 + (80)^2 + (2)^2}$

$= 1538.08$ units.

Stream function:

It is defined as the scalar function of space and time such that it's partial derivative with respect to any direction given the velocity component at right angles to that direction. It is denoted by "ψ" and defined only for two dimensional flow. Mathematically for steady flow. It is defined as.

$$\psi = f(x,y).$$

$$\frac{\partial \psi}{\partial x} = v \; ; \; \frac{\partial \psi}{\partial y} = -u.$$

The continuity equation for two dimensional flow is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

$$\frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial x}\right) = 0.$$

$$-\frac{\partial^2 \psi}{\partial x \partial y} + \frac{\partial^2 \psi}{\partial x \cdot \partial y} = 0.$$

Hence existence of 'ψ' means a possible case of fluid flow. The flow may be rotation and irrotational. The rotational component

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$$\boxed{\omega_z = \frac{1}{2}\left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right]}$$

when $\omega_z = 0 \Rightarrow \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \rightarrow$ Irrotational.

$$\boxed{\omega_z = 0}$$

v) velocity potential function:

It is defined as a scalar function of space & time such that it's -ve' derivative with respect to any direction gives the fluid velocity in that direction. It is defined by ϕ. Mathematically the velocity potential is defined as $\phi = f(x,y,z)$ for steady flow. Such that

$$u = -\frac{\partial \phi}{\partial x} \; ; \; v = -\frac{\partial \phi}{\partial y} \; ; \; w = -\frac{\partial \phi}{\partial z} \quad \text{—— (1).}$$

where u, v, w are the components of velocity in (x,y,z) directions. The continuity equation for an incompressible steady flow is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.$$

Sub. the values of u, v, w in a above Equation

$$\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial y}\right) + \frac{\partial}{\partial z}\left(-\frac{\partial \phi}{\partial z}\right) = 0.$$

$$-\frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial y^2} - \frac{\partial^2 \phi}{\partial z^2} = 0.$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \text{ — laplace equation}$$

for two dimensional case

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0.$$

The velocity potential (ϕ) is given by an expression $\phi =$
$\phi = -xy^3/3 - x^2 + \frac{x^3 y}{3} + y^2$ find i) velocity components in x & y directions
ii) S.T. ϕ represents a possible case of flow.

Sol: Given,

$$\phi = -\frac{xy^3}{3} - x^2 + \frac{x^3 y}{3} + y^2$$

$$\frac{\partial \phi}{\partial x} = \frac{\partial}{\partial x}\left(-\frac{xy^3}{3} - x^2 + \frac{x^3 y}{3} + y^2\right)$$

$$= \left(-\frac{y^3}{3} - 2x + \cancel{3}\frac{x^2 y}{\cancel{3}}\right) = \left(-\frac{y^3}{3} - 2x + x^2 y\right)$$

$$\frac{\partial \phi}{\partial y} = \frac{\partial}{\partial y}\left(-\frac{xy^3}{3} - x^2 + \frac{x^3 y}{3} + y^2\right) = -\frac{\cancel{3}xy^2}{\cancel{3}} - 0 + \frac{x^3}{3} + 2y.$$

$$u = -\frac{\partial \phi}{\partial x} = -\left(-\frac{y^3}{3} - 2x + x^2 y\right) = \frac{y^3}{3} + 2x - x^2 y \text{ — (1)}$$

$$v = -\frac{\partial \phi}{\partial y} = -\left(-xy^2 + \frac{x^3}{3} + 2y\right) = xy^2 - \frac{x^3}{3} - 2y \text{ — (2)}$$

ii) The given value of ϕ will represents a possible case of flow if it satisfy the laplace Eqn.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0.$$

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{y^3}{3} + 2x - \frac{3x^2 y}{3}\right) = 2 - 2xy.$$

$$\frac{\partial^2 \phi}{\partial y^2} = \frac{\partial}{\partial y}\left(xy^2 - \frac{x^3}{3} - 2y\right) = 2xy - 2.$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0.$$

$$\cancel{2} - \cancel{2xy} - \cancel{2} + \cancel{2xy} = 0$$

$$0 = 0.$$

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Equipotential line:-

A line along which potential ϕ is constant is known as Equipotential line.

ϕ = constant.

But $\phi = f(x,y)$ for steady flow.

Then $d\phi = \frac{\partial \phi}{\partial x} \cdot dx + \frac{\partial \phi}{\partial y} \cdot dy$.

where $\frac{\partial \phi}{\partial x} = -u \quad \frac{\partial \phi}{\partial y} = -v$.

$d\phi = -u\,dx - v\,dy$

$d\phi = -(u\,dx + v\,dy)$

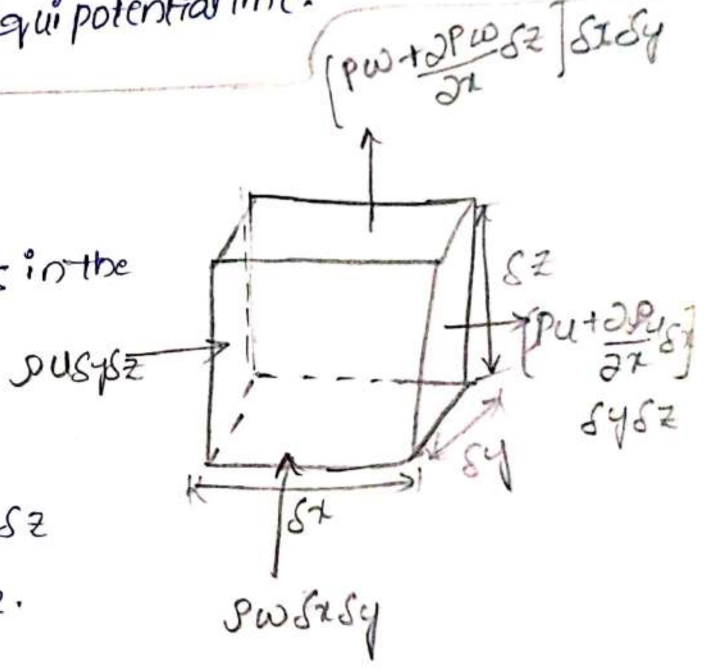
for equipotential line $d\phi = 0$.

$u\,dx + v\,dy = 0$

$u\,dx = -v\,dy$

$\frac{u}{v} = -\frac{dy}{dx}$.

∴ Hence it is a slope of equipotential line.



$M_t = \rho\, \delta x\, \delta y\, \delta z$

The time rate of change of mass in the control volume is.

$\frac{\partial \rho}{\partial t} \delta x \delta y \delta z$

The net flow on x-direction.

$\left(\rho u + \frac{\partial \rho u}{\partial x}\delta x\right)\delta y \delta z - \rho u \delta y \delta z$

$= \frac{\partial \rho u}{\partial x} \delta x \delta y \delta z$.

The net flow on y-face is

$\frac{\partial \rho v}{\partial y} \delta x \delta y \delta z$

The net flow on z-face is.

$\frac{\partial \rho w}{\partial z} \cdot \delta x \delta y z$.

Adding all the net flow and dividing by the volume $(dx\,dy\,dz)$.

The continuity equation in cartesian co-ordinates.

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0.}$$

Date :
8-2-21

UNIT IV

Fluid Dynamics

Dynamic :
The study of fluids which is motion by considering the forces in fluids.

Forces:- Forces are two types Surface force and body force.

Equations of motion:-
According to newtons second law of motion of the net force 'F_x' acting on a fluid element in the direction of 'x' and is equal to mass(m) of the fluid element multiplied by a acceleration 'a_x' in the x-direction.

$$F_x = m \times a_x \quad \text{—} (1)$$

In the fluid flow the following forces are present

F_g = forces due to gravity.
F_p = force due to pressure
F_v = force due to viscosity.
F_z = force due to turbolents.
F_c = force due to compressibility.

Thus the Eqn (1)

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_z)_x + (F_c)_x$$

If the force due to compressibility 'F_c' is neglessible then the resulting net force equal to

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_z)_x$$

For flow where 'F_z' is neglessible then the reading Eqn of motion are known as "Navier- strokes equation"

$$(F_x) = (F_g)_x + (F_p)_x + (F_v)_x$$

If the flow is assumed to be ideal, viscous force 'F_v' is zero Eqn of motion are known as "Eulers" Equation of motion.

$$F_x = (F_g)_x + (F_p)_x.$$

Flow net:- A grid obtained by drawing a series of equipotential lines and stream lines is called a "flow net". The flow net is an important tool in annalysing two dimensional irrotational flow problems.

Vertex flow:-
vertex flow is defined as the flow of a fluid along a curved path (Or) the flow of a rotating mass of fluid is known as 'vertex flow'.

i) free vertex flow ii) force vertex flow.

Free vertex flow:-

when no external is required to rotate the fluid mass that type of flow is called of free vertex flow.

The relation b/w velocity & radius in free vertex is obtain by putting the value of external torque is equal to zero.

consider a fluid partical of mass m' at a radial distance 'r' from the axis of rotation, having a tangential velocity 'v'.

Then angular momentum $= m \times v$

moment of momentum= moment $\times r = m \times v \times r$.

$\therefore$ Time rate of change of angular momentum= $\frac{\partial}{\partial t}(mvr)$.

for free vertex flow $\frac{\partial}{\partial t}(mvr) = 0$.

on intigrating $\int \frac{\partial}{\partial t}(mvr) = 0$.

$mvr =$ constant.

$$vr = \frac{\text{constant}}{m} = \text{constant}.$$

Forced vertex flow:-

Forced vertex flow is defined as the type of vertex flow in which some external torque is required to rotate the fluid mass. The fluid mass in this type of flow rotates at constant angular velocity 'ω'. The tangential velocity of any fluid partical is given by $\boxed{v = \omega r}$

where;

r = radius of fluid partical.

hence angular velocity

$$\boxed{\omega = \frac{v}{r} = \text{constant}}$$

A 250 litres of water is flowing in a pipe having a diameter of 300mm. the pipe is bent at 135° find the magnitude and direction of resultant force for the bend end, the pressure of water flowing is 39.24 N/cm².

Sol: Given,

$Q = 250 lit = 250 \times 10^{-3} m^3/sec$

$d = 300mm = 0.3m.$

$\theta = 135°$

$p = 39.24 N/cm^2$

$= 39.24 \times 10^4 N/m^2$

$Q = A_1 V_1 = A_2 V_2$

$A_2 = A_1 = \frac{\pi}{4} \times (0.3)^2 = 0.0706 m^2.$

$V_2 = \frac{Q}{A} = \frac{250 \times 10^{-3}}{0.0706} = 3.54 m/sec.$

force acting on the bent in x & y - direction. ($\because V_1 = 0$)

$F_x = \rho Q (V_2 - V_1 \cos\theta) + P_1 A_1 - P_2 A_2 \cos\theta.$

$= 1000 \times 250 \times 10^{-3} (3.54) + 39.24 \times 10^4 \times 0.0706) (1 - \cos 135°)$

$F_x = 48.19 \times 10^3 N.$

$F_y = \rho Q (-V_2 \sin\theta) - P_2 A_2 \sin\theta$

$= 1000 \times 250 \times 10^{-3} (-3.54 \sin 135°) - 39.24 \times 10^4 \times 0.0706 \times \sin 135°.$

$F_y = -20.21 \times 10^3 N.$

Resultant:

$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{(48.17)^2 + (20.21)^2}$

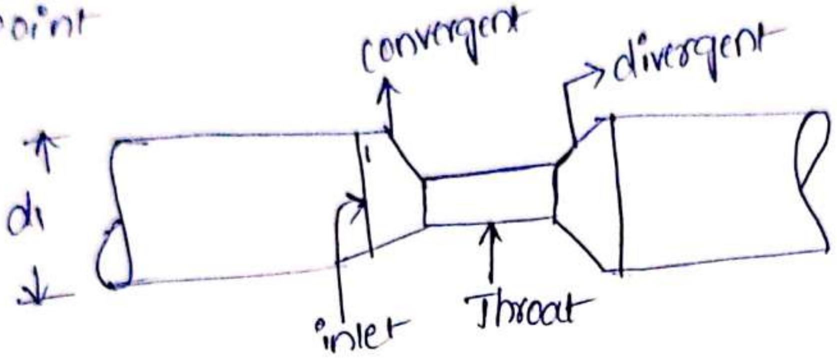
$= 52.23 \times 10^3 N$

$\tan\theta = \frac{F_y}{F_x}$

$\theta = \tan^{-1}\left(\frac{20.21}{48.17}\right) = 22° 45'$

Venturimeter:

* venturimeter, pitot tube, orifice meter are working on the principle of Bernoulli's equation.
* venturimeter & orifice meter is used to measure rate of flow (Q)
* Pitot tube is used to measure velocity at a particular point



Equation for C_d :-

* consider a venturimeter fixed to a horizontal pipe. venturimeter having at section ①-① $P_1 A_1 V_1$ & $P_2 A_2 V_2$ at section ②-②

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 \qquad (\because Z_1 = Z_1)$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} \qquad \left[\because \frac{P_1 - P_2}{\rho g} = \text{pressure difference} = h\right]$$

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$$

$$\frac{h}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$$

Discharge $Q = a_1 V_1 = a_2 V_2$

$$V_1 = \frac{a_2 V_2}{a_1}$$

$$h = \frac{V_2^2 - \frac{a_2^2 V_2^2}{a_1^2}}{2g}$$

$$h = \frac{V_2^2}{2g} - \frac{\left(\frac{a_2 V_2}{a_1}\right)^2}{2g}$$

$$h = \frac{V_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right]$$

$$h = \frac{V_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

$$V_2^2 = \frac{2gh \times a_1^2}{a_1^2 - a_2^2}$$

$$V_2 = \frac{\sqrt{2gh} \times a_1}{\sqrt{a_1^2 - a_2^2}}$$

$\therefore Q_{th} = a_2 V_2$

$$Q_{th} = \frac{a_2 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}} \quad \boxed{\therefore Q_{act} = Q_{th} \times C_d} \quad [\because C_d < 1 \text{ always}]$$

C_d for venturimeter varies from 0.95 to 0.98

orificimeter 0.62 to 0.65

Case i):-

let the differential manometer contains heavier liquid than flowing liquid.

let;

S_h = sp. gr of heavier liquid

S_0 = sp. gr of liquid flowing through the pipe.

x = difference of the heavier column in U-tube

then

$$h = x\left[\frac{S_h}{S_0} - 1\right]$$

Case ii):- liquid in on manomer lighter than flowing liquid.

$$h = x\left[1 - \frac{S_l}{S_0}\right] \quad S_l = \text{sp.gr of lighter manometer.}$$

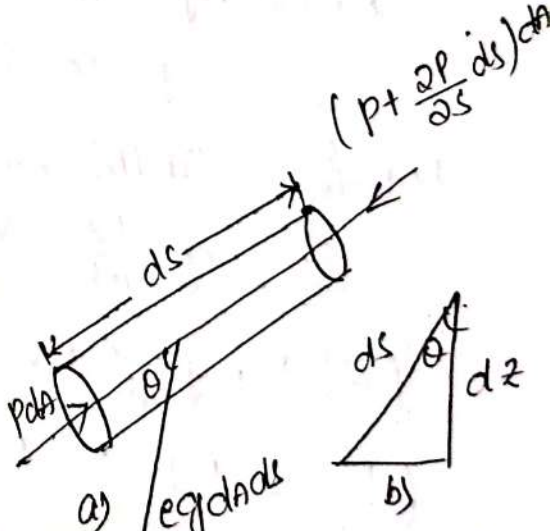
Case iii):- venturimeter is placed inclined & differential U-tube manometer

$$h = \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) \quad \left[\because \frac{V^2}{2g} \text{ is constant}\right]$$

$$h = x\left[\frac{S_h}{S_0} - 1\right]$$

Case iv):- $h = x\left[1 - \frac{S_l}{S_0}\right]$

Euler's Equation of motion:-



This is the Equation of motion in which forces due to gravity and pressure are taken to be considered. This is derived by considering the motion of a fluid element in a stream line.

Consider a stream line in which flow is taking place in 's' direction as shown in fig. Consider a cylindrical element of c/s pxdA and length ds. The forces acting on the cylindrical element are

pressure force PdA in the direction of flow.

pressure flow $(P + \frac{\partial P}{\partial S} ds) dA$ opposite to the direction of flow.

$\therefore$ wt. of the element = $\rho g\, dA\, ds$.

Let 'θ' is the angle b/w the direction of flow and line of action of the wt. of element.

The resultant force on the fluid element in the direction of 's' must be equal to the mass of fluid into acceleration in the direction 's'.

$$\text{Resultant force}(F) = P\,dA - \left(P + \frac{\partial P}{\partial s} ds\right) dA - \rho g\, dA\, ds \cos\theta$$
$$= \rho\, dA\, ds \times a_s \quad \text{— (1)}$$

where 'a_s' is the acceleration $a_s = \frac{dv}{dt}$, where 'v' is the function of s & t.

$$= \frac{\partial v}{\partial s} \cdot \frac{ds}{dt} + \frac{\partial v}{\partial t} \quad \left[\because \frac{ds}{dt} = v\right]$$

$$a_s = v \cdot \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}.$$

If the flow is steady then $\frac{\partial v}{\partial t} = 0$

$$a_s = v \cdot \frac{\partial v}{\partial s}.$$

Sub. the value of a_s in eqn (1) & simplify eqn we get,

$$P\,dA - P\,dA - \frac{\partial P}{\partial s} \cdot ds\, dA - \rho g\, dA\, ds \cos\theta = \rho\, dA\, ds \times v \cdot \frac{\partial v}{\partial s}.$$

$$-\frac{\partial P}{\partial s} ds\, dA - \rho g\, dA\, ds \cos\theta = \rho\, dA\, ds \times v \cdot \frac{\partial v}{\partial s}.$$

Divide by $\rho\, dA\, ds$ on both sides.

$$\frac{-\frac{\partial P}{\partial s}}{\rho} - g\cos\theta - v \cdot \frac{\partial v}{\partial s} = 0.$$

but from fig (b) we have $\cos\theta = \frac{dz}{ds}$.

$$\frac{-\frac{\partial P}{\partial s}}{\rho} - g\frac{dz}{ds} - v \cdot \frac{dv}{ds} = 0$$

$$-\left[\left(\frac{\frac{\partial P}{\partial s}}{\rho}\right) + g\frac{dz}{ds} + v \cdot \frac{\partial v}{\partial s}\right] = 0.$$

$$-\left(\frac{1}{\rho}\frac{\partial P}{\partial s} + g \cdot \frac{dz}{ds} + v \cdot \frac{\partial v}{\partial s}\right) = 0.$$

$$\frac{1}{\rho}\, dp + g \cdot dz + v \cdot dv = 0.$$

$$\frac{dP}{\rho} + g \cdot dz + v \cdot dv = 0.$$

This Equation is known as Euler's equation of motion.

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Bernoulli's equation from Euler's equation:

Bernoulli's Equation is obtained by integrating the Euler's eqn of motion.

$$\frac{dP}{\rho} + g\,dz + v\,dv = 0.$$

$$\int \frac{dP}{\rho} + \int g\,dz + \int v\,dv = \int 0 = \text{constant}.$$

$$\frac{1}{\rho}\int dP + g\int dz + \int v\,dv = \text{constant}.$$

If flow is in incompressible 'ρ' is constant then

$$\frac{1}{\rho}P + g.z + \frac{v^2}{2} = \text{constant}.$$

$$\frac{P}{\rho} + g.z + \frac{v^2}{2} = \text{constant}.$$

divided by 'g' on both sides.

$$\frac{P}{\rho g} + z + \frac{v^2}{2g} = \text{constant}.$$

$$\boxed{\frac{P}{\rho g} + \frac{v^2}{2g} + z = \text{constant}.}$$

$\frac{P}{\rho g}$ = pressure energy per unit wt. of fluid (or) pressure head.

$\frac{v^2}{2g}$ = kinetic energy per unit wt. (or) kinetic head.

z = potential energy per unit wt. (or) potential head.

This is known as Bernoulli's equation.

Assumptions:-

* The following are the assumptions made in the derivation of bernoulli's equation.

* The fluid is ideal, viscosity is zero.
* The fluid is steady.
* The fluid flow is incompressible.
* The fluid flow is irrotational.

(ii) Momentum equation:-

It is based on the law of conservation of momentum (or) on the momentum principle, which states that the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in the direction. The force acting on a fluid mass (m) is given by the Newton's second law of motion.

$$F = ma$$
$$F = m\left(\frac{dv}{dt}\right)$$
$$F.dt = m(dv)$$
$$F\,dt = d(mv).$$

It is the momentum principle. which is known as impulse momentum equation and states that the impulse of a force 'F' acting on a fluid of mass m' in a short interval of time 'dt' is equal to the change of momentum 'd(mv)' in the direction of force. The impulse momentum equation is used to determine the resultant force exerted by a flowing fluid on a pipe bent. consider two sections ① & ②. as shown in fig:

V_1, P_1, A_1 = velocity, pressure, Area at ①.

V_2, P_2, A_2 = velocity, pressure, Area at ②.

let F_x & F_y be the component of the forces exerted by the flowing fluids on the bend in x & y directions respectively. Then the force exerted by the flowing fluid bend on the fluid in the directions x & y will be equal to $-F_x$ & $-F_y$ but in the opposite directions. Hence component of the force exerted by the bend on the fluid in the x-direction = $-F_x$. The other external forces acting on a fluid are P_1A_1 & P_2A_2 on the sections ① & ② respectively. Then the momentum eqn in x-direction is given by

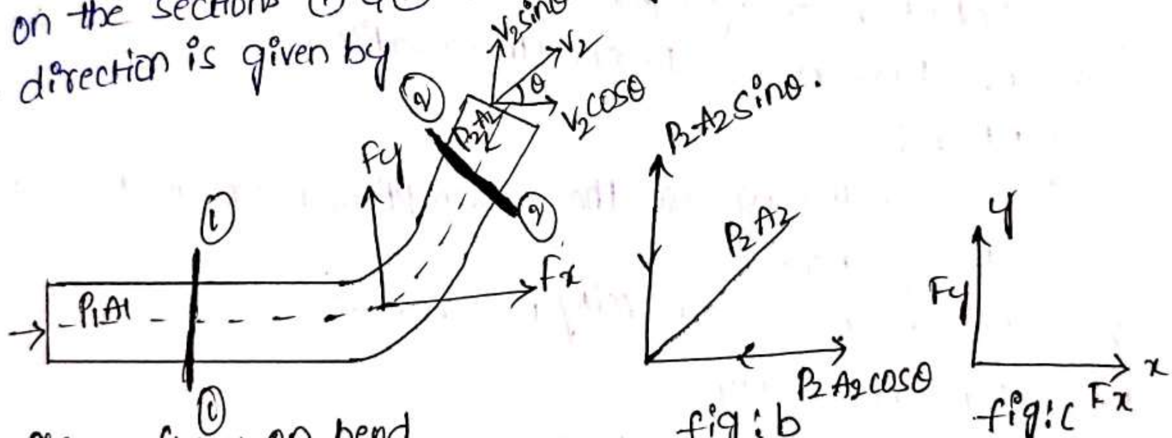


fig: a forces on bend fig: b fig: c

Net forces acting on fluid in the direction of 'x' = mass force acting

= mass per second × Change of velocity.

$$P_1A_1 - P_2A_2\cos\theta - F_x = \rho Q.(\text{final velocity} - \text{initial velocity}).$$

$$P_1A_1 - P_2A_2\cos\theta - F_x = \rho Q(V_2\cos\theta - V_1)$$

$$P_1A_1 - P_2A_2\cos\theta - \rho Q(V_2\cos\theta - V_1) = F_x.$$

$$\boxed{F_x = \rho Q(V_1 - V_2\cos\theta) + P_1A_1 - P_2A_2\cos\theta.}$$

fluid flow in x-direction.

Similarly momentum equation in y-direction

$$0 - P_2 A_2 \sin\theta - F_y = \rho Q (V_2 \sin\theta - 0)$$

$$[\because V_1 = 0]$$

$$-P_2 A_2 \sin\theta - \rho Q (V_2 \sin\theta) = F_y.$$

$$\boxed{F_y = \rho Q (V_2 \sin\theta) + P_2 A_2 \sin\theta.}$$

[$\because$ '−' is neglected]

Resultant force$(F_R) = \sqrt{(F_x)^2 + (F_y)^2}$

$$\boxed{\therefore \tan\theta = \frac{F_y}{F_x}.}$$

Angle made by the resultant force with horizontal direction.

Problems on venturimeter:-

① A horizontal venturimeter with inlet on ~~throught~~ throat diameter 30cm & 15cm respectively it is used to measure the flow of water. The reading of manometer connected to the inlet & throat is 20cm of mercury. Determine the rate of take $C_d = 0.98$

Sol: Given,

Diameter of Inlet $(d_1) = 30cm = 0.3m$.

Diameter of throat $(d_2) = 15cm = 0.15m$.

$C_d = 0.98$.

$A_1 = \frac{\pi}{4} \times d_1^2 = \frac{\pi}{4} \times (0.3)^2 = 0.0706 m^2$

$A_2 = \frac{\pi}{4} \times d_2^2 = \frac{\pi}{4} \times (0.15)^2 = 0.0176 m^2$.

Pressure difference $x = 20cm = 0.2m$

$$h = x\left[\frac{S_h}{S_0} - 1\right] \Rightarrow 0.2\left[\frac{13.6}{1} - 1\right]$$

$$= 2.52m$$

$$Q = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{(0.076) \times (0.0176)}{\sqrt{(0.076)^2 - (0.0176)^2}} \times \sqrt{2 \times 9.81 \times 2.52}$$

$$= 0.125 \, m^3/sec$$

(or) $Q = 125$ lit/sec.

② A horizontal venturimeter with inlet on throat 20cm & 10cm is used to measure the flow of oil of sp. gr. 0.8. The discharge of oil through venturimeter is 60 lit/sec. Find the reading of oil, mercury in differential manometer Take $C_d = 0.98$.

Sol: Given,

$D_1 = 20cm = 0.2m$ $\qquad$ $Q = C_d$

$D_2 = 10cm = 0.1m$.

$Q = 60$ lit/sec $= 60 \times 10^{-3} m^3/sec$.

Sp gravity of oil $= 0.8$

$C_d = 0.98$

$a_1 = \frac{\pi}{4} \times d^2 = \frac{\pi}{2} \times (20)^2 = 314.159 \times 10^{-3} m^2$

$$a_2 = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 10^2 = 78.539 \times 10^{-3}\ m^2.$$

$$Q = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}.$$

$$60 \times 10^{-3} = \frac{0.98 \times 314.159 \times 10^{-3} \times 78.539 \times 10^{-3} \times \sqrt{2 \times 9.81 \times h}}{\sqrt{(314.159 \times 10^{-3})^2 - (78.539 \times 10^{-3})^2}}$$

$$60 \times 10^{+3} = 0.079449 \times 4.429 \sqrt{h}.$$

$$\sqrt{h} = \frac{60 \times 10^{+3}}{0.035209} \qquad \sqrt{h} = 170381.93.$$

$$\text{s.o.b.s} \Rightarrow h = 2.906.$$

$$h = 2.90$$

$$h = x\left[\frac{S_h}{S_o} - 1\right]$$

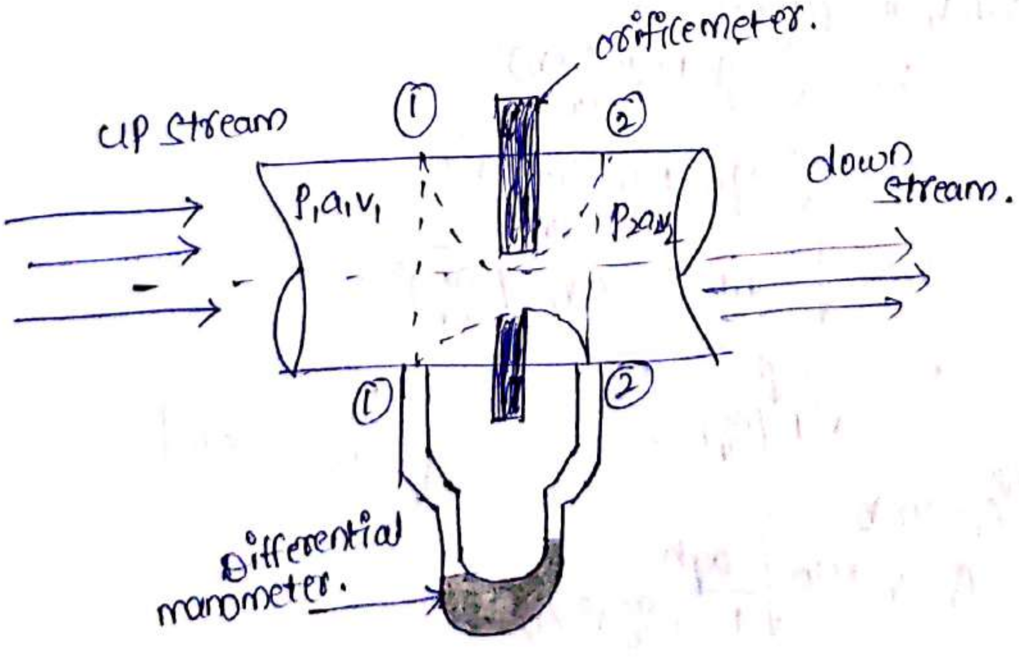
$$2.906 = x\left[\frac{13.6}{0.8} - 1\right]$$

$$x = 0.181\ m$$

$$\text{or} \quad x = 18.162\ cm.$$

Orifice meter:-

It is a device used for measuring rate of flow of a fluid through a pipe. It is cheeper device compare to venturimeter. It also works on the same principle as that of venturimeter & Bernoulli's principle. It consists of a circular plate which has a circular sharp edge hole called "orifice". which is concentric with pipe. The orifice diameter is generally kept 0.5 times than the pipe though it may vary from 0.4 to 0.8 times.



Applying Bernoulli's eqn at section (1) & (2).

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 .$$

$z_1 = z_2$.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} = h \text{ (differential head)}$$

$z_1 \neq z_2$.

$$\left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = h.$$

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$V_2^2 = V_1^2 - 2gh$$

$$V_2 = \sqrt{V_1^2 - 2gh}$$

$$V_2 = \sqrt{2gh + V_1^2} \quad \text{——— (1).}$$

Now at section (2) vena contracta

$a_0 = A_2 =$ represents of vena contracta.

we have $C_c = \frac{a_0}{a_2}$

where $a_2 = C_c \times a_0$

By using continuity equation

$$a_1 V_1 = a_2 V_2$$

$$V_1 = \frac{a_2 V_2}{a_1}$$

$$V = \frac{C_c \times a_0 V_2}{a_1}$$

imp.

$$\boxed{C_d = C_c \times C_v}$$

C_d = co-efficient of discharge.

C_c = co-efficient of contraction.

C_v = co-efficient of velocity.

Sub 'V_1' in eq (1), we get

$$V_2 = \sqrt{2gh + \left(\frac{a_0 C_c \times V_2}{a_1}\right)^2}$$

$$V_2 = \sqrt{2gh + \frac{C_c^2 \times a_0^2 \times V_2^2}{a_1^2}}$$

$$V_2 = \sqrt{2gh + C_c^2 \times V_2^2 \left(\frac{a_0}{a_1}\right)^2}$$

$$V_2 = \sqrt{\frac{2gh}{1 - \left(\frac{a_0}{a_1}\right)^2 \times C_c^2}}$$

The discharge

$$Q = a_2 V_2 \qquad [\because a_2 = C_c \times a_0]$$

$$Q = C_c \times a_0 \sqrt{\frac{2gh}{1 - \left(\frac{a_0}{a_1}\right)^2 \times C_c^2}}$$

$$Q = C_c \times a_0 \sqrt{\frac{2gh}{1-\left(\frac{a_0}{a_1}\right)^2 \times C_c^2}} \times \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}$$

$$Q = C_c \times a_0 \times \frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 \times C_c^2}} \times \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}$$

$$Q = C_c \times a_0 \times \frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 \times C_c^2}} \times \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}$$

$$\boxed{Q = Cd\,\frac{a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}}$$

① An orifice meter with orifice dia 10cm is inserted in a pipe of dia 20cm, the pressure guages fitted upstream & downstream of the orifice meter gives reading of 19.62 N/cm^2 & 9.81 N/cm^2. co-efficient of discharge of an orifice meter 0.60.. find the discharge of water through pipe.

Sol: Given,

Orifice meter $d_0 = 10cm$

pipe diameter $d_1 = 20cm$.

Area $(A_1) = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (10)^2 = 78.53 cm^2$.

Area $(A_2) = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (20)^2 = 314.15 cm^2$.

$P_1 = 19.62\, N/cm^2$

$P_2 = 9.81\, N/cm^2$.

$h = \frac{P_1 - P_2}{\rho g} = \frac{19.62 - 9.81}{1000 \times 9.81} = 1000cm$ of water & 10m of water.

$$Q = Cd.\frac{a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}} = \frac{0.6 \times 78.53 \times 314.15 \times \sqrt{2 \times 9.81 \times 1000}}{\sqrt{(314.15)^2 - (78.53)^2}}.$$

$Q = 68.172$ Lit/sec.

② An orifice meter with orifice dia 15cm is inserted in a pipe of 30cm diameter. The pressure difference measured by mercury, oil in differential manometer on two sides of the orifice meter gives a reading of 50cm of mercury Find the rate of flow of oil of sp.g. 0.9 when Cd = 0.64.

Sol: Given,

Dia of orifice $d_o = 15\,cm$

$\therefore$ Area $a_o = \pi/4\,(15)^2 = 176.7\,cm^2$

Dia of pipe $d_1 = 30\,cm$

$a_1 = \pi/4\,(30)^2 = 706.85\,cm^2$

Sp. gr. of oil $S_o = 0.9$

Difference $(x) = 50\,cm$ of mercury.

$\therefore$ Differential head $h = x\left[\frac{S_g}{S_o} - 1\right] = 50\left[\frac{13.6}{0.9} - 1\right]$ cm of oil.

$= 50 \times 14.11 = 705.5$ cm of oil.

$C_d = 0.64$

The rate of the flow, Q is given Equation

$$Q = C_d \frac{a_o a_1}{\sqrt{a_1^2 - a_o^2}} \times \sqrt{2gh}$$

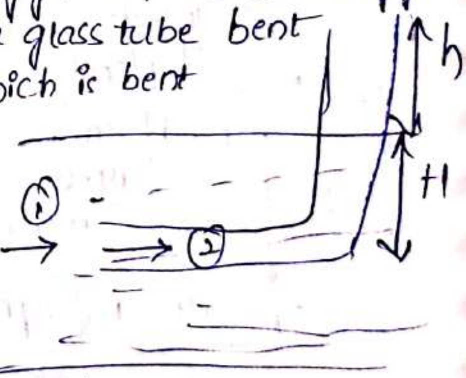
$$Q = 0.64 \times \frac{176.7 \times 706.85}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 9.81 \times 1000}$$

$$Q = 137414.25\ cm^3/sec$$

$$Q = 137.41425\ lit/sec.$$

Pitot tube:-

* It is a device used for measuring the ~~vot~~ velocity of flow at any point in a pipe (or) channel. It is based on the ~~pressure there~~ principle that the velocity of flow at a point becomes 'zero', the pressure there is increased due to convertion of kinetic energy into pressure energy. In it's simplest form the pitot tube consist of a glass tube bent at right angles as show in fig. The lower end which is bent through 90° is directed in the upstream direction. The liquid rises up in the tube due to convertion of kinetic energy into pressure energy. The velocity is determined by measuring the rise of fluid in the tube.



pitot tube.

Consider two points ① & ② at the same level in such a way that point ② is just as the inlet of the pitot tube and point ① is for away from the tube.

Let;

P_1 = Intensity of pressure at point ①

V_1 = velocity of flow at point ①

P_2 = pressure at point ②.

V_2 = velocity at point (2).

H = Depth of tube in the liquid.

h = Rise of liquid in the tube above the free surface.

Applying Bernoulli's eqn at points (1) & (2).

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$z_1 = z_2$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\frac{P_1}{\rho g} = H \ ; \ \frac{P_2}{\rho g} = h + H.$$

imp.

$$\boxed{C_d = C_c \times C_v.}$$

$$H + \frac{V_1^2}{2g} = (h+H) + \frac{V_2^2}{2g}.$$

$$\frac{V_1^2}{2g} = h + \frac{V_2^2}{2g}.$$

$$V_1^2 = \left(h + \frac{V_2^2}{2g}\right) 2g \qquad V_1 = \sqrt{2gh + V_2^2}$$

[V_2 is neglected].

$$V_1 = \left(\frac{2gh + V_2^2}{2g}\right)$$

$$\boxed{V_1 = C_v \sqrt{2gh}}$$

$$\therefore V_1 = \sqrt{2gh}$$

$$\boxed{V_{th} = \sqrt{2gh} \ ; \ V_{act} = C_v \sqrt{2gh}} \qquad \boxed{C_d = C_v \times C_c}$$

Pitot static tube:-

Pitot static tube which consists of two circular concentric tubes one inside the other with some annular space in between as shown in fig. The outlet of these two tubes are connected to the differential manometer where the direcn difference of pressure head 'h' is measured by knowing the difference of levels of manometer liquid say 'x' then

$$\boxed{h = x\left[\frac{S_g}{S_o} - 1\right]}$$

(1) A pitot static tube placed in the centre of 300mm of pipe as one orifice pointing upstream side & the other ⊥er to it. The velocity in the pipe is 0.8 of the central velocity. Find the discharge through that pipe if the pressure difference b/w two orifice is 60mm. Take the $C_V = 0.98$ for pitot tube.

Sol: Given,

Diameter (d) = 300mm

velocity at its centre $V_1 = 0.8$ m/sec

pressure difference x = 60 mm = 0.06m. = h.

$$\text{Mean velocity} = 0.8 \times V_1(act)$$
$$= 0.8 \times C_V \sqrt{2gh}$$
$$= 0.8 \times 0.98 \sqrt{2 \times 9.81 \times 0.06}.$$
$$= 0.85 \text{ m/sec}.$$

$$\text{Discharge }(Q) = A_1V_1$$
$$= 0.0706 \times 0.85$$
$$= 0.06 \text{ m}^3/\text{sec}.$$

(2) Find the velocity of flow of an oil through a pipe when the difference of mercury level in a different U-tube manometer connected to the tapering of the pitot tube is 100mm. Take $C_V = 0.98$ & sp.gr. 0.8.

Sol: Given,

Difference of mercury level x = 100mm = 0.1m

sp. gravity of oil $S_0 = 0.8$

sp. gravity of mercury $S_g = 13.6$.

$$h = x\left[\frac{S_g}{S_0} - 1\right] = 0.1\left[\frac{13.6}{0.8} - 1\right]$$

$$\boxed{h = 1.6.}$$

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$V = C_v \sqrt{2gh}$

$= 0.98 \times \sqrt{2 \times 9.81 \times 1.6}$

$\boxed{\therefore V = 5.490 \ m/sec}$

Problems on Bernoulli's Equation

① water is flowing through a pipe of 5cm diameter under a pressure of 29.43 N/cm² and with mean velocity of 2m/sec. Find the total pressure (or) Total energy per unit wt. of the water at a c/s which is 5M above the datum line.

Sol: Given,

Diameter of pipe $d = 5cm = 0.5M$.

pressure $= 29.43 \ N/cm^2$.

$= 29.43 \times 10^4 \ N/m^2$.

Velocity $(V) = 2m/sec$.

datum head $= 5m$.

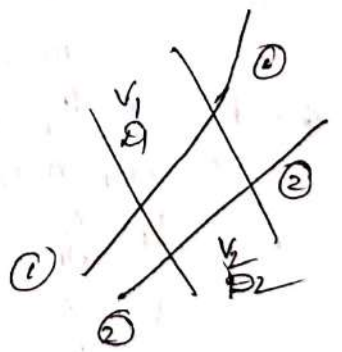
Total head = Pressure head + velocity head + Datum head.

$$= \frac{P}{\rho g} + \frac{V^2}{2g} + z.$$

$$= \frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{2^2}{2 \times 9.81} + 5.$$

Total head $= 35.20m$.

② A pipe through which water is flowing, having a dia 20cm and 10cm at the c/s ① & ② respectively. The velocity of water at section ① is 4m/sec. find the velocity head at section ① & ② and also rate of discharge.



Sol: Given,

$D_1 = 20cm = 0.2M$

$D_2 = 10cm = 0.1m$

$A_1 = \frac{\pi}{4} \times (0.2)^2 = 0.03141 m^2$.

$A_2 = \frac{\pi}{4} \times (0.1)^2 = 0.007853 m^2$.

$V_1 = 4m/sec$.

from continuity Equation

$A_1 V_1 = A_2 V_2$

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{0.03141 \times 4}{0.007853}$$

$= 15.9989 \ m/sec$.

$\therefore$ Discharge $= A_1V_1 = 0.03141 \times 4 = 0.12564 \ m^3/sec$.

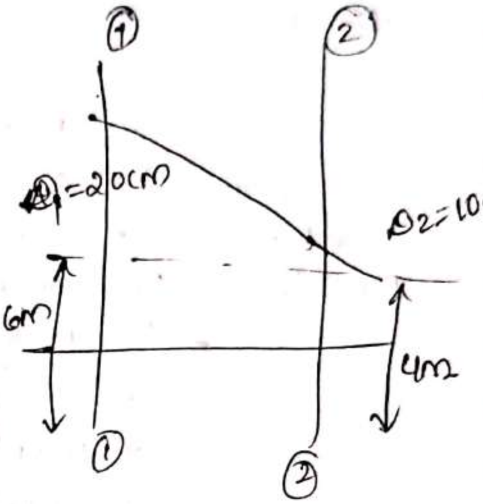
Velocity head

Section ① $= \frac{V_1^2}{2g} = \frac{(4)^2}{2\times9.81} = 0.8154$.

Section ② $= \frac{V_2^2}{2g} = \frac{(15.9989)^2}{2\times9.81} = 13.0461$

③ The water flowing through a pipe having dia 20cm & 10cm at section ① & ② respectively. The rate of flow through pipe is 35 lit/sec. at the section ① is 6m above the datum & section ② is 4m above the datum. If the pressure at the section ① 39.24N/cm². Find the intensity of pressure at section ②.

Sol: Given;



Section ①:

$D_1 = 20cm = 0.2m$

$A_1 = \frac{\pi}{4} \times (0.2)^2 = 0.03141 m^2$.

$P_1 = 39.24 \ N/cm^2$.

$= 39.24 \times 10^4 \ N/m^2$.

$Z_1 = 6m$.

Section ②:

$D_2 = 10cm = 0.1m$.

$A_2 = \frac{\pi}{4} \times (0.1)^2 = 0.007853 m^2$.

$Z_2 = 4m$.

$P_2 = ?$

Rate of flow $Q = 35$ lit/sec.

$= 35 \times 10^{-3} \ m^3/sec$.

According to continuity equation

$Q = A_1V_1 = A_2V_2$

$V_1 = \frac{Q}{A_1} = \frac{35\times10^{-3}}{0.03141} = 1.114 \ m/sec$.

$V_2 = \frac{Q}{A_2} = \frac{35\times10^{-3}}{0.0078} = 4.45 \ m/sec$.

According to Bernoulli's Equation

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{39.24\times10^4}{1000\times9.81} + \frac{(1.114)^2}{2\times9.81} + 6 = \frac{P_2}{1000\times9.81} + \frac{(4.45)^2}{2\times9.81} + 4.$$

$$46.066 = \frac{P_2}{9.81 \times 1000} + 5.0093$$

$$P_2 = (46.066 - 5.0093) \times (9.81 \times 1000)$$

$$\boxed{\therefore P_2 = 40.27 \times 10^4 \ N/m^2}$$

UNIT-V

Analysis of pipe flow.

Energy losses in pipe flow.

Major losses

This due to friction & it is calculated by

1. Darcy weisbach formula.
2. Chezy's formula.

Minor losses.

1. Sudden contraction
2. Sudden expansion
3. Bent pipe
4. pipe fittings.
5. An abstruction in pipe.

when a fluid is flowing through a pipe the fluid experiance some resistance due to which some of the energy of the fluid is losed. This loss of energy is classification as above flow chart.

Loss of energy due to friction:-

1. Darcy weisbach formula:-
This loss of head (or) energy in pipes due to friction is calculated by Darcy weisbach formula

$$\boxed{h_f = \frac{4fLV^2}{2g \times d}}$$ V.V.imp.

where; h_f = loss of head due to friction.

f = co-efficient of friction which a function of reynolds number

$= \frac{16}{Re}$ for $Re < 2000$

$\frac{0.079}{(Re)^{1/4}}$ for $Re > 4000$.

L = length of pipe; D = Dia of pipe; V = mean velocity of flow.

2. Chezy's formula:- loss of head due to friction in a pipe.

$$\boxed{h_f = \frac{f'}{eg} \times \frac{P \times L \times V^2}{A}} \quad \text{— (1)}$$

h_f = loss of head due to friction.
f = co-efficient of friction.
P = wetted perimeter of pipe.
L = length of pipe.
V = velocity of flow.
A = Area of c/s of pipe.

Hydraulic mean depth (m) $= \frac{A}{P} = \frac{\frac{\pi}{4}d^2}{\pi d} = \frac{d}{4}$

substituting $\frac{A}{P}$ with m.

$$h_f = \frac{f'}{\rho g} \times \frac{1}{m} \times L V^2$$

$$V^2 = \frac{h_f \times \rho g \times m}{f' \times L}$$

$$V = \sqrt{\frac{h_f \times \rho g \times m}{f' \times L}}$$

$$V = \sqrt{\frac{\rho g}{f'}} \times \sqrt{\frac{h_f \times m}{L}} \qquad C = \sqrt{\frac{\rho g}{f'}} \;;\; i = \sqrt{\frac{h_f}{L}}.$$

$$V = C \times \sqrt{\frac{h_f \times m}{L}} \qquad \text{where; } \sqrt{\frac{h_f}{L}} = i.$$

$\boxed{\therefore V = C \times \sqrt{mi}}$ The value of 'm' for pipe is always d/4.

(1) An oil of sp.gr. 0.7 is flowing through a pipe of dia 300mm @ of 500 lit/sec. Find the head loss due to friction at power required to maintain the flow for a length of 1000 meters. Take kinematic viscosity $\vartheta = 0.29$ strokes.

Sol: Given,

Sp. gravity of oil = 0.7 ; Density (ρ) = $0.7 \times 1000 = 700$.

Dia of pipe $d = 300\,mm = 0.3\,m$.

Area $= \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (0.3)^2 = 0.0706\,m^2$

Discharge (Q) = 500 lit/sec = $500 \times 10^{-3}\,m^3/sec = 0.5\,m^3/sec$.

length (l) = 1000m

kinematic viscosity (ϑ) = 0.29 strokes.

imp.

$$\therefore Re = \frac{\rho V D}{\mu} \text{ (or) } Re = \frac{VD}{\vartheta}$$

$$V = \frac{Q}{A} = \frac{0.5}{0.0706} = 7.08\,m/sec.$$

Reynold's number $Re = \frac{V \cdot D}{\vartheta} = \frac{7.08 \times 0.3}{0.29 \times 10^{-4}}$.

$$= 7.317 \times 10^4$$

co-efficient of friction $(f) = \frac{0.079}{(Re)^{1/4}} = \frac{0.079}{(7.317 \times 10^4)^{1/4}}$

$$\boxed{f = 0.0048}$$

$h_f = \frac{4fLv^2}{2gd}$

$= \frac{4 \times 0.0418 \times 1000 \times (7.08)^2}{2 \times 9.81 \times 0.3}$

$h_f = 163.5$

power required $= \frac{\rho \times g \times Q \times h_f}{1000}$

$= \frac{700 \times 9.81 \times 0.5 \times 163.5}{1000}$

$= 561.37$ watts.

v.v. imp.

(2) water is flowing through a pipe of dia 200 mm with a velocity 3 m/sec find the head loss due to friction for a length of 5 meters. If the co-efficient of firction is given by $f = 0.02 + \frac{0.09}{(Re)^{0.3}}$ where 'Re' is Reynolds n.o. Take $\nu = 0.01$ strokes.

Sol: Given,

Dia $d = 200mm = 0.2m$.

$v = 3$ m/sec.

$(L) = 5m$.

$f = 0.02 + \frac{0.09}{(Re)^{0.3}}$.

kinematic viscosity $\nu = 0.01$ strokes $= 0.01 \times 10^{-4}$ m²/sec.

$Re = \frac{3 \times 0.2}{0.01 \times 10^{-4}} = 6 \times 10^5$

$f = 0.02 + \frac{0.09}{(6 \times 10^5)^{0.3}}$.

$f = 0.0216$

Head loss due to friction $h_f = \frac{4fLv^2}{2gd} = \frac{4 \times 0.0216 \times 5 \times 3^2}{2 \times 9.81 \times 0.2}$

$\boxed{h_f = 0.993m}$

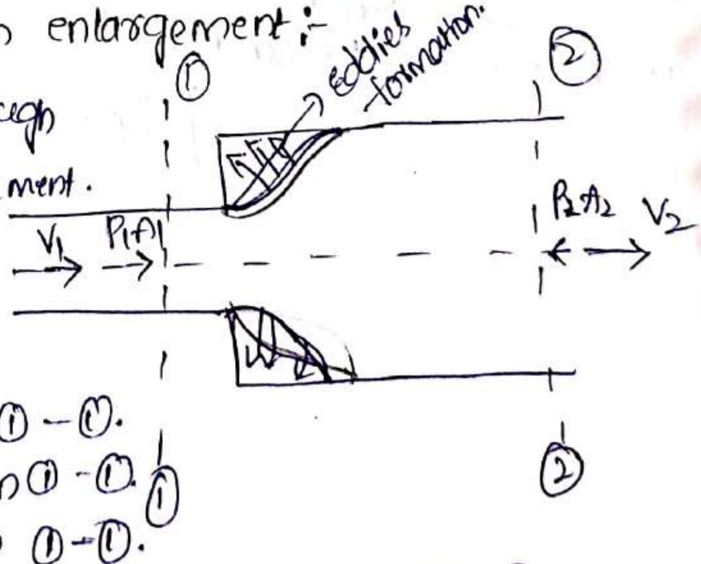
Minor energy head losses:

The loss of head (or) energy due to friction in a pipe is known as major loss while the loss of energy due to change of velocity of the full flowing fluid in magnitude (or) direction is called minor loss of energy the minor loss of energy includes the following cases.

* loss of head due to sudden enlargement.
* loss of head due to sudden contraction.
* loss of head at the entrance of pipe.
* loss of head at exit of the pipe.
* loss of head due to an obstruction in a pipe.
* loss of head due to bend in the pipe.
* loss of head in various pipe fittings.

In case of long pipe the above losses are small as compared with the loss of head due to friction and hence they are called minor losses. And even may be neglected without serious error. But in case of short pipe these losses are comparable with the the loss due to friction.

loss of head due to sudden enlargement:-



consider a liquid flowing through a pipe which has sudden enlargement. consider two sections ①-① & ②-② before and after enlargement.

let;

P_1 = pressure intensity at section ①-①.

V_1 = velocity of flow at section ①-①.

A_1 = Area of pipe at section ①-①.

P_2, V_2, A_2 are corresponding values at section ②-②.

Due to sudden change of diameter of the pipe from d_1 to d_2 the liquid flowing from the smaller pipe is not able to follow the abrupt change of the boundary. Thus the flow separates from the boundary and turbulent eddies are formed the loss of head takes place due to formation of eddies. eddies are formed flow of water and touches the corner places forms a algey. That algey is known as "Eddies".

p' = pressure intensity of the liquid eddies on the area $(A_2 - A_1)p'$.

h_e = loss of head due to sudden enlargement.

Applying Bernoullies Equation at section ①-① & ②-②.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 \ (h_e) + h_e.$$

$$Z_1 = Z_2.$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_e$$

$$h_e = \left(\frac{P_1 - P_2}{\rho g}\right) + \left(\frac{V_1^2 - V_2^2}{2g}\right) \quad \text{——} ①$$

consider the control volume of the liquid between section ①-① and ②-②. Then the force acting on the liquid in the controll volume in the direction of flow is given by

$$F_x = P_1A_1 + P'(A_2 - A_1) - P_2A_2$$

$$p' = P_1$$

$$F_x = P_1\cancel{A_1} + P_1A_2 - P_1\cancel{A_1} - P_2A_2$$

$$F_x = P_1A_2 - P_2A_2$$

$$F_x = (P_1 - P_2)A_2 \quad \text{——} ②$$

Momentum of liquid per second at section ①-①.

Momentum = mass x velocity.

$$= \rho \times A_1V_1 \times V_1$$

$$= \rho \times A_1 \times V_1^2.$$

at section ②-② $= \rho \times A_2 V_2^2$

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Change of momentum per second $= \rho A_2 V_2^2 - \rho A_1 V_1^2$.

From continuity $A_1 V_1 = A_2 V_2$

$$A_1 = \frac{A_2 V_2}{V_1}$$

$$= \rho A_2 V_2^2 - \rho \times \frac{A_2 V_2}{V_1} \times V_1^{\cancel{2}}$$

$$= \rho A_2 V_2^2 - \rho \times A_2 V_2 \times V_1$$

$$= \rho A_2 V_2 \left[V_2 - V_1\right].$$

$$= \rho A_2 \left[V_2^2 - V_2 V_1\right] \text{ — (3).}$$

Now net force acting on the control volume in the direction of flow must be equal to the rate of change of momentum. The equation is given below.

Equating (2) & (3) & dividing with 'g' on both sides.

$$\frac{(P_1 - P_2)\cancel{A_2}}{g} = \frac{\rho \cancel{A_2}\left[V_2^2 - V_2 V_1\right]}{g}$$

$$\frac{P_1 - P_2}{\rho g} = \frac{\left[V_2^2 - V_2 V_1\right]}{g} \text{ — (4).}$$

Substitute in Eqn (1).

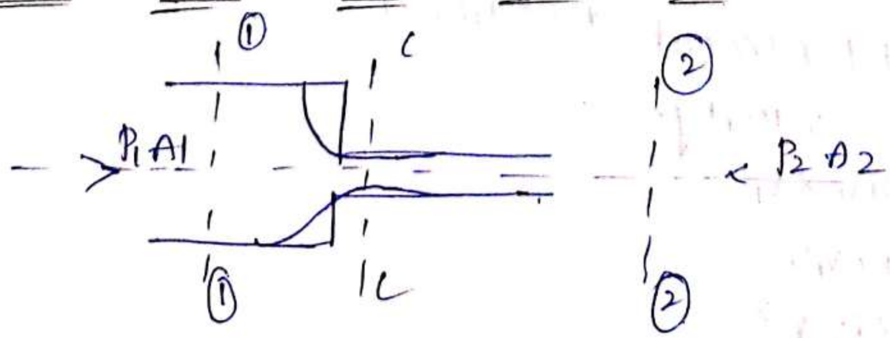
$$h_e = \frac{\left[V_2^2 - V_2 V_1\right]}{g} + \left[\frac{V_1^2 - V_2^2}{2g}\right]$$

$$h_e = \frac{2V_2^2 - 2V_2 V_1 + V_1^2 - V_2^2}{2g}$$

$$h_e = \frac{V_2^2 + V_1^2 - 2V_2 V_1}{2g}$$

$$\boxed{h_e = \frac{(V_1 - V_2)^2}{2g}}$$

Loss of head due to Sudden contraction:-



consider a liquid flowing in a pipe which has a sudden contraction in area as shown in fig:

consider (2) sections (1)-(1) & (2)-(2) before and after contraction as the liquid flows from a large pipe to a smaller pipe the area of flow those on decreasing and becomes min. at section c-c. A sudden enlargement of the area takes place the loss of head due to sudden contraction is actually due to sudden enlargement from vena-contracta to smaller pipe.

let,

A_c = Area of flow at section c-c.

V_c = velocity of flow at section c-c.

A_2 = Area of flow at section (2)-(2).

V_2 = velocity of flow at section (2)-(2).

h_c = loss of head due to sudden contraction.

Now;

h_c = actual i.e. loss of head due to enlargement from section c-c to section (2)-(2) is given by

$$h_c = \frac{(V_c - V_2)^2}{2g} = \frac{V_2^2}{2g}\left(\frac{V_c}{V_2} - 1\right) \text{ —— (1)}$$

from continuity eqn.

$$A_c V_c = A_2 V_2$$

$$\frac{V_c}{V_2} = \frac{A_2}{A_c} = \frac{1}{A_c/A_2} = \frac{1}{C_c}$$

substitute the value $\frac{V_c}{V_2}$ in (1).

$$h_c = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right) \qquad \left[\because K = \frac{1}{C_c} - 1\right]$$

$$\boxed{h_c = K\left(\frac{V_2^2}{2g}\right)}$$

'C_c' range from assumed to be 0.62 then

$$K = \left(\frac{1}{0.62} - 1\right)^2$$
$$= 0.375.$$

If the value of 'C_c' is not given then the head loss due to contraction is taken as $\left(0.5\frac{V_2^2}{2g}\right)$.

Loss of head at entrance of the pipe:-

This is due loss of energy which occurs when a liquid enters a pipe which is connected to a large tank (or) Reservoir. This loss is similar to the loss of head due to contraction. This loss depends on the form of entrance. For a sharp edge entrance. This loss is more than a rounded (or) Bell mouthed entrance. In practice the value of loss of head at the entrance of a pipe with sharp cornered entrance is taken as '$0.5 \frac{V^2}{2g}$' where 'V' is the velocity of liquid in pipe.

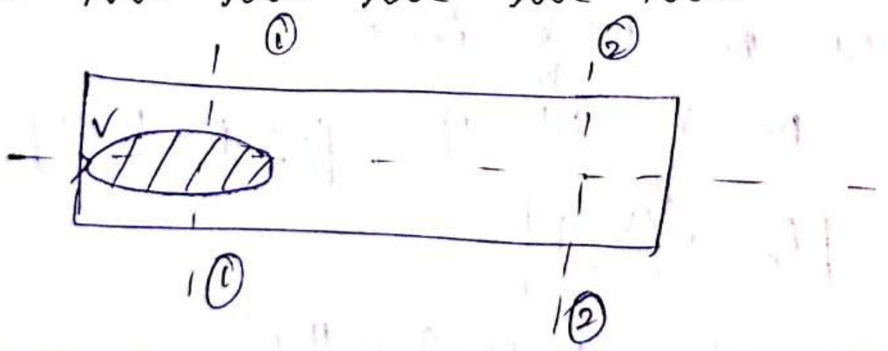
This loss is denoted with $\boxed{h_i = 0.5 \frac{V^2}{2g}}$

loss of head at exit of the pipe:-

This is the loss of head due to the velocity of liquid at outlet of the pipe which is dissipated either in the form of a free jet (if outlet of the pipe is free) (or) it is lost in the tank (or) Reservoir. This loss is equal to $\frac{V^2}{2g}$ where 'V' is the velocity of liquid at outlet of pipe this is denoted by

$$\boxed{h_0 = \frac{V^2}{2g}}.$$

Loss of head due to an obstruction in a pipe:-



When ever there is an obstruction in a pipe the loss of energy takes place due to reduction of the area of the c/s of the pipe at the place where obstruction is present. There is a sudden enlargement of the area of flow beyond the obstruction due to which loss of head takes place as shown in fig.

consider a pipe of area of C/S, it is having an obstruction let,

a = Max. area of obstruction.
A = Area of pipe.
V = velocity of liquid in pipe.

Then (A - a) = Area of flow of liquid at section (1) - (1).

As the liquid flows and passes through section (1) - (1) a vena contracta is found beyond section (1) - (1) after which the stream of liquid wides again and velocity of flow at section (2) - (2) becomes uniform and equal to velocity.

'v' in a pipe. This suituation is similar to the flow of liquid through sudden enlargement.

V_c = velocity of liquid at vena contracta.

Then the loss of head due to obstruction = loss of head due to enlargement.

$$\frac{(V_c - V)^2}{2g} \text{ --- (1).}$$

from continuity eqn.

$$a_c \times V_c = A \times V \text{ --- (2).}$$

$$C_c = \frac{\text{Area of vena contracta}}{(A - a)}$$

$$C_c = \frac{a_c}{A - a}.$$

$$\therefore a_c = C_c(A - a).$$

sub 'a_c' in (2).

$$C_c(A - a) \times V_c = A \times V.$$

$$V_c = \frac{A \times V}{C_c(A - a)}$$

sub 'V_c' in Eq (1).

$$\frac{\left(\left[\frac{A \times V}{C_c(A - a)}\right] - V\right)^2}{2g}.$$

$$\boxed{\frac{V^2}{2g}\left[\frac{A}{C_c(A - a)} - 1\right]^2}$$

Hydraulics And Hydraulic Machinery

Date:- 15-4-21

UNIT-I

Laminar & Turbulent Flow in Pipes.

The laminar flow is disscused in previous chapter. The laminar flow the fluid particals move along straight parallel paths in layers & laminae such that paths of individual f.p. donot cross their neighbouring particals. laminar flow is possible at low velocities when the fluid is highly viscous. But when velocity increase & less viscous the fluid particals donot move in straight paths. The fluid particall move in random manner resulting in general mixing of the particals. These type of flow turbulent flow.

A laminar flow changes to turbulent flow when velocity is increase & dia of pipe is increase & viscosity of fluid decrease. O Reynold is was first to demonstrate transition from laminar to turbulent not only on the mean velocity but on the quantity $\frac{\rho V D}{\mu}$. This quantity is a dimensionless quantity & is called Reynold N.o (Re). In the case of circular

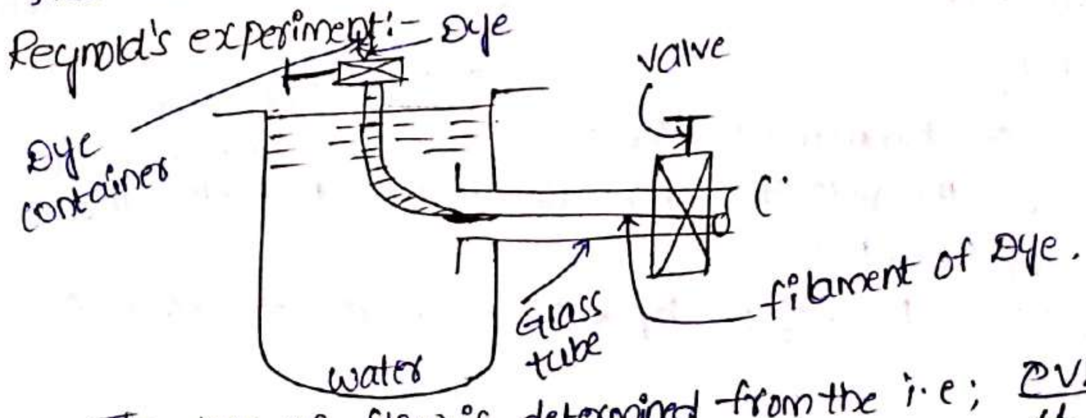
$Re < 2000$ – laminar flow

$Re > 4000$ – Turbulent flow

$2000 < Re < 4000$ – Transition. flow.

Thus the flow changes from laminar to turbulent.

Reynold's experiment:-

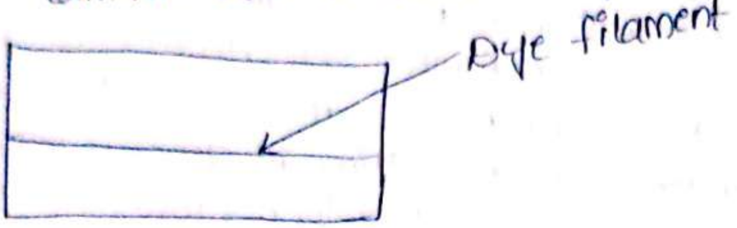


The type of flow is determined from the i.e; $\frac{\rho V D}{\mu}$ this was demonstrate by OReynold in 1883. His apparatus consist of.

1. A tank containing water at constant.
2. A small tank, contain some Dye.
3. A glass tube having a well mouthed entrance at one end & a regulating valve at other end.
4. The water from the tank was allowed to follow through glass tube. The velocity of fluid

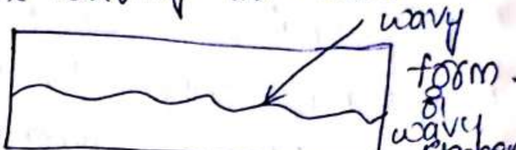
A liquid dye having same specific weight as water was introduced into the glass tube as shown in figure.
The following observations were made by Reynold.

1. when the velocity of flow was low the Dye filament in the glass tube was in the form of a straight line. This straight line was parallel to glass tube. which is a case of laminar flow as shown in fig.



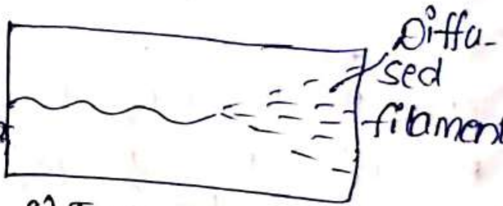
a) laminar flow

2. with the increase of velocity of flow the dye-filament was no longer a straight line but it became waving this shows that flow is no longer laminar.



b) Transition flow

3. with further increase of velocity the wavy dye filament broke up and finally diffuced this means that the fluid particles of the dye at higher velocity are moving in random fashion which shows the case of turbulent flow.
Thus in the case of turbulent flow mixing of turbulent & water is flow irregular and dis orderly.

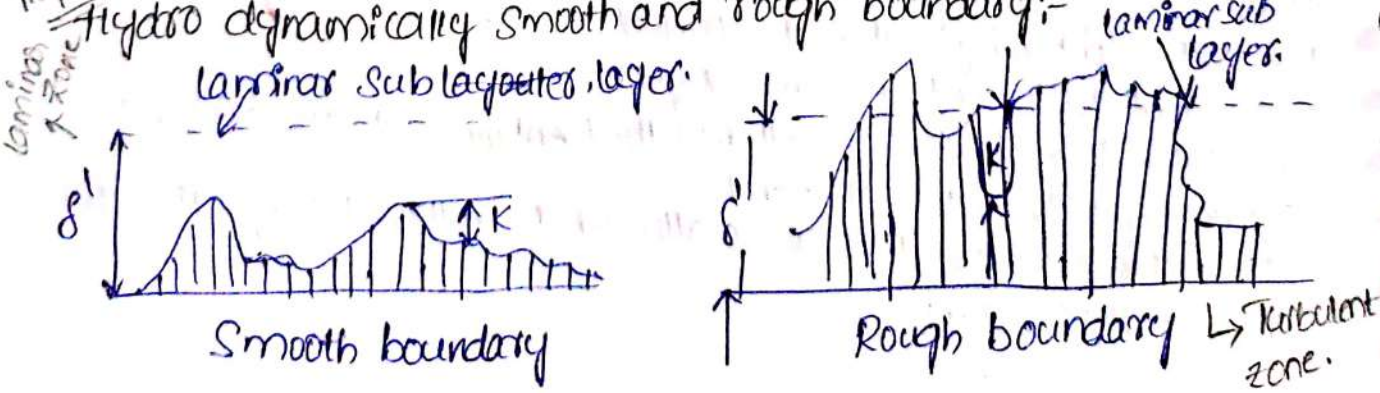


c) Turbulent flow

In case of laminar flow the loss of pressure head was found to be proportional the velocity but in case of turbulent flow loss of los head is approximately
more exactly loss of head hf is $h_f \propto V^n$ where 'n' varies from 1.75 to 2.

Date:- 17-4-21

imp Hydro dynamically smooth and rough boundary:-



Smooth boundary

Rough boundary

let 'k' is the average height of the irregularities projecting from the surface of boundary as shown in fig. If the value of k large for boundary is rough. If the value of k is less then the boundary is smooth boundary. This is the classification of rough & smooth boundary on boundary characteristics. But proper characterising the flow & fluid characteristics are also to be consider. $k < \delta'$ - smooth boundary.

k is greater than δ' - Rough boundary. $(k > \delta')$.

* If k by δ' is less than 0.25 than the boundary is smooth.
* $\frac{k}{\delta'} > 6.0$ boundary is rough.
* If $0.25 < \frac{k}{\delta'} < 6.0$ - Transition.
* In terms of roughness Reynold's number $\frac{uk}{v}$.
* If $\frac{uk}{v} < 4$ - smooth bounday.
* If $\frac{uk}{v}$ lies b/w 4 & 100 then it transition stage $\frac{uk}{v} > 100$ - rough.

imp.

Velocity distribution for turbulent flow in smooth pipes:-

The velocity distribution in smooth & rough pipe is given by eqn $u = \frac{u^*}{k} \log_e y + c$. It may be seen that at $y = 0$. The velocity u is $-\infty$ (infinity). This means that velocity 'u' is +ve at some distance far away from the wall & $-\infty$. Hence at some finite distance from wall the velocity will be zero. let, this distance from pipe valve is y'. Now the constant 'c' is determine from the boundary condition i.e; at $y = y'$, $u = 0$

Hence the eqn becomes $0 = \frac{u^*}{k} \log_e y' + c$ $\boxed{c = \frac{-u^*}{k} \log_e y'}$ y- distance

Sub 'c' in first eqn

$$u = \frac{u^*}{k} \log_e y - \frac{u^*}{k} \log_e y' \Rightarrow \boxed{\frac{u^*}{k}\left[\log_e y/y'\right] = u.}$$

Sub the value of $k = 0.4$.

$$u = \frac{u^*}{0.4}\left[\log_e y/y'\right] \therefore u = 2.5 u^* \log_e (y/y')$$

$$\log_e (y/y') = 2.3 \log_{10} (y/y').$$

$$\frac{u}{u^*} = 2.5 \times 2.3 \log_{10} (y/y')$$

$$\frac{u}{u^*} = 5.75 \log_{10} (y/y').$$

For the smooth boundary there exist a laminar sublayer as shown in f. The velocity distribution in laminar sublayer is parabolic in nature. The velocity distribution does hold good. Thus, it In the laminar s.l. log velocity distribution can be assumed that y' assumed to δ'. δ' = thickness of laminar sublayer.

$y' \alpha \delta'$.

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Then the value of y' is given as $y' = \frac{\delta'}{107}$; $\delta' = \frac{11.6V}{u*}$

V = Kinematic viscosity.

$$y' = \frac{11.6V}{u*} \times \frac{1}{107} = \frac{0.108V}{u*}$$

Sub y' in above eqn.

$$\frac{u}{u*} = 5.75 \log_{10} y/\left(\frac{0.108V}{u*}\right).$$

$$\frac{u}{u*} = 5.75 \log_{10} \frac{u*y}{V} + 5.55 . \quad \boxed{\therefore U = 5.75 \log_{10} \frac{U*y}{0.108V} + 5.55}$$

Date :- 19-4-2021

Velocity distribution for Turbulent flow in rough pipes:-

$$\frac{U}{U*} = 5.75 \log_{10} \left[\frac{y}{\frac{k}{30}}\right]$$

$$= 5.75 \log_{10} \left[\frac{y}{k}\right] \times 30$$

$$= 5.75 \log_{10} \left(\frac{y}{k}\right) + 5.75 \log_{10} (30.0)$$

$$\boxed{\frac{U}{U*} = 5.75 \log_{10} \left(\frac{y}{k}\right) + 8.5 .}$$

(1) A pipe line carrying water has the avg height of irregularities projecting from the surface of the boundary of the pipe has 0.15mm. what type of boundary is it. The shear stress developed is 4.9 N/m². The Kinematic viscosity of water is 0.01 strokes.

Sol: Given,

K = 0.15mm avg height of irregularities

$= 0.15 \times 10^{-3}$ m.

Shear stress developed $\tau_0 = 4.9 N/m^2$

Kinematic viscosity $\nu = 0.01$ strokes $= 0.01 \times 10^{-4} m^2/sec$.

Density of water $\rho = 1000 kg/m^3$.

$\therefore$ Shear velocity $u_x = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{4.9}{1000}} = 0.07$ m/sec.

Roughness Reynolds number $= \frac{U_x K}{\nu} = \frac{0.07 \times 0.15 \times 10^{-3}}{0.01 \times 10^{-4}}$

$= 10.5$

Since $\frac{U_x K}{V}$ lies in b/w 4 & 100 and pipe surface behaves as in transition.

(2) A rough pipe is of diameter 8cm. The velocity at a point 3cm from wall is 30% more than the velocity at a point 1cm from pipe wall. Determine the avg height of roughness.

Sol: Given,

Dia of rough pipe = 8cm = 0.08m

let velocity of flow at 1cm from pipe wall = u.

The velocity of flow at 3cm from pipe wall: 1.3u

The velocity distribution for rough pipe is given by equation

$$\frac{u}{u_*} = 5.75 \log_{10}\left(\frac{y}{k}\right) + 8.5 \quad ; k = \text{Height of roughness.}$$

for a point 1cm from pipe wall we have

$$\frac{u}{u_*} = 5.75 \log_{10}\left(\frac{1.0}{k}\right) + 8.5 \text{ --- (1)}$$

for a point 3cm from pipe wall we have velocity is 1.3u

$$\frac{1.3u}{u_*} = 5.75 \log_{10}\left(\frac{3.0}{k}\right) + 8.5 \text{ --- (2)}$$

Divide (2) by (1), we get

$$\frac{\frac{1.3\cancel{u}}{\cancel{u_*}}}{\frac{\cancel{u}}{\cancel{u_*}}} = \frac{5.75 \log_{10}\left(\frac{3.0}{k}\right) + 8.5}{5.75 \log_{10}\left(\frac{1.0}{k}\right) + 8.5} \Rightarrow 1.3 = \frac{5.75 \log_{10}\left(\frac{3.0}{k}\right) + 8.5}{5.75 \log_{10}\left(\frac{1.0}{k}\right) + 8.5}$$

$$7.475 \log_{10}\left(\frac{1}{k}\right) + 11.05 = 5.75 \log_{10}\left(\frac{3}{k}\right) + 8.5.$$

$$7.475 \log_{10}\left(\frac{1}{k}\right) - 5.75 \log_{10}\left(\frac{3}{k}\right) = 8.5 - 11.05.$$

11.05
8.5
.55

$$7.475(\log_{10} 1 - \log_{10} k) - 5.75(\log_{10} 3 - \log_{10} k) = -2.55$$

$$7.475(0 - \log_{10} k) - 5.75(0.477 - \log_{10} k) = -2.55$$

$$-7.475 \log_{10} k - 2.74 + 5.75 \log_{10} k = -2.55$$

$$-7.475 \log_{10} k + 5.75 \log_{10} k = -2.55 + 2.74$$

$$-1.725 \log_{10} k = 0.1933$$

$$\log_{10} k = \frac{-0.1933}{1.725}$$

$$\boxed{\therefore k = 0.89\text{cm.}}$$

Date:-
20-4-21

Boundary layer:- Concept of boundary layer was first introduced by L. Prandtyl in 1904. when a real fluid flows through a solid boundary a layer of fluid which comes in contact to the boundary & sticks to it & the condition of non-slip occurs when velocity of fluid is equal to the velocity of the boundary. ↳ velocity becomes zero.

* velocity of flowing fluid increases rapidly from 0 at the boundary surface and approaches the maximum velocity.
* The layer adjacent to the boundary is known as boundary layer.
* Boundary layer is formed when there is a relative motion between the fluid and the boundary layer.

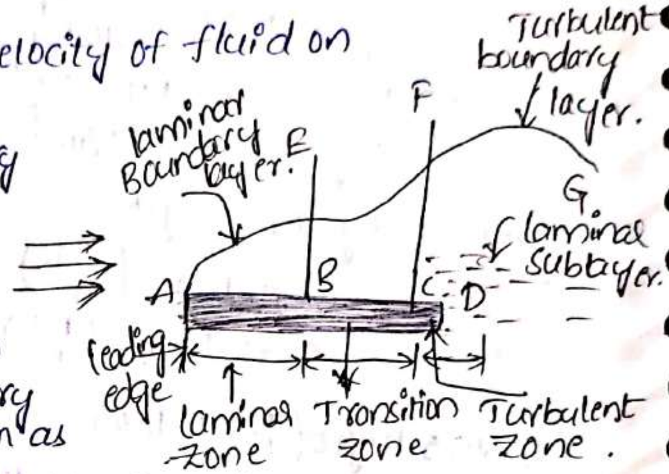
* Fluid exerts shear force on the boundary layer and boundary layer exerts equal and opposite shear resistance force on the fluid.

Regions of Boundary:-

1. A thin layer adjoining the boundary is called boundary layer. Shear stress is present. $\tau = \mu \times \frac{du}{dy}$.
2. The region out side the boundary layer where velocity is constant shear stress is zero. Since velocity gradient $\frac{du}{dy} = 0$.

Laminar boundary layer:-

* consider a stationary plate and hence velocity of fluid on surface of the plate is zero.
* After some distance from the boundary layer fluid attains some velocity.
* Boundary layer starts from leading edge.
* Near the leading edge where the flow thickness is small, the flow in the boundary layer is laminar. This layer is known as laminar boundary layer.



* The length upto which the laminar boundary layer exists is known as laminar zone. This distance is equal to 5×10^5 for a plate according to renold's number. $\boxed{(Re)_x = \frac{U \times x}{\nu}}$ ν = Kinematic viscosity of fluid.

x = dist from leading edge; U = free from stream velocity of fluid

$$\boxed{5 \times 10^5 = \frac{U \times x}{\nu}} \quad (2)$$

Turbulent Boundary layer:-

* If the length of the plate is more from the equation (2) then the thickness of the boundary layer will go on increasing.
* laminar flow is distributed and irregular which leads to transition from laminar to turbulent boundary layer.
* The zone in which laminar changes to turbulent boundary layer.
* After transition zone thickness of the layer increases known as Turbulent zone.

Laminar Sub layer:- This is the region in the turbulent boundary layer zone adjacent to the solid surface. Hence the velocity variation influenced by viscuos effects.

Boundary layer thickness :- (δ).

Distance from the boundary of the solid body measured in the y-direction to the point where the velocity of the fluid is appox equal to 0.99 times the free stream velocity U.

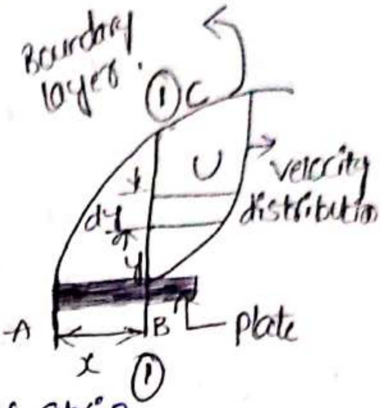
Displacement thickness (δ^*):

The distance perpendicular to the boundary by which the free stream is displaced due to the formation of boundary layer.

consider the flow velocity equal to 'U' over a plate. At a distance 'x' from leading section. consider section ① - ①.

At 'B' velocity is zero. At 'c' velocity is 'U'.

Distance BC = δ.



let;

y = distance of elemental strip from the plate.

dy = thickness of elemental strip

U = velocity of fluid at the elemental strip.

b = width of the plate.

Area of the elemental strip dA = b×dy.

mass/sec flowing through the strip = $\rho \times v \times$ Area of strip.

$= \rho \times u \times b \times dy$ — ①.

mass/sec flowing through strip $= \rho \times U \times b \times dy$ — ②.

If 'U' is more than u. Reduction in mass/sec flowing through the strip $= \rho \times U \times b dy - \rho \times u \times b dy$.

$= \rho \times b \times dy\,(U-u)$.

∴ Total reduction in mass of fluids through 'BC' due to plate.

$= \int_0^{\delta} \rho \times b \times dy\,(U-u) = \rho \times b \int_0^{\delta} (U-u)\,dy$ — ③.

If δ^* is the distance displaced then loss of mass of the fluid per second flowing through the distance δ^* is.

$= \rho \times V \times A$.

$= \rho \times U \times \delta^* \times A$ — ④.

$\rho \times b \int_0^{\delta} (U-u)\,dy = \rho \times U \times \delta^* \times b$

$\int_0^{\delta} (U-u)\,dy = U \times \delta^*$

$\delta^* = \int_0^{\delta} \left(\frac{U-u}{U}\right) dy$.

$$\boxed{\delta^* = \int_0^{\infty} \left(1 - \frac{u}{U}\right) dy}$$

Momentum thickness (θ):-

Distance measured perpendicular to the boundary of the solid body by which the boundary should be compensate for the reduction in momentum of the flowing fluid on account of boundary layer formation. It is denoted by 'θ'.

momentum of fluid = mass × velocity. ⇒ $(\rho \times u \times b dy) \times u$.

$= \rho \times b \times dy \times u^2$ — ①

Momentum of the fluid in the absence of boundary layer.

$= (\rho \times u \times b \times dy)\,U$ — ②

Loss of mometum through elemental strip.

② - ① ⇒ $(\rho \times u \times b dy)U - (\rho \times b dy \times u^2)$

$= \rho \times b dy \times u\,(U-u)$ — ③

Total loss of momentum per second.

$$= \int_0^{\delta} \rho \times b \times dy \times u (U-u).$$

$$= \rho \times b \int_0^{\delta} (U-u) u\, dy \text{ --- } (4).$$

let θ = distance by which plate is displaced when the fluid is flowing with a constant velocity U.

= Mass of fluid through θ x velocity.

= ρ x area x velocity x velocity.

$= (\rho \times \theta \times b \times U) \times U.$

$= \rho \times \theta \times b \times U^2$ --- (5).

(4) = (5).

$$\rho \theta b U^2 = \int_0^{\delta} \rho b u (U-u) dy.$$

$$\rho \theta b U^2 = \rho b u \int_0^{\delta} (U-u) dy$$

$$\theta = \frac{1}{U^2} \int_0^{\delta} (U \times u - u^2) dy \Rightarrow \theta = \int_0^{\delta} \left(\frac{u}{U} - \frac{u^2}{U}\right) dy.$$

$$\theta = \int_0^{\delta} \frac{1}{U} \left(u - \frac{u^2}{U}\right) dy. = \int_0^{\delta} \frac{u}{U} \left(1 - \frac{u}{U}\right) dy.$$

Energy thickness (δ^{**}):-

To reduce K.E.

Distance measured perpendicular to the boundary of the solid body by which boundary should be displaced to compensate the reduction in KE of the flowing fluid on account of boundary layer formation and is denoted by δ^{**}.

Kinetic energy of the fluid $= \frac{1}{2} mv^2 = \frac{1}{2} (\rho \times u \times b \times dy) \times u^2$ --- (1).

Kinetic energy of the fluid in the absence of boundary layer $= \frac{1}{2} (\rho \times u \times b dy) U^2$ --- (2).

K.E of the fluid in the strip. (2) - (1).

loss of K.E. through strip.

$= \frac{1}{2} (\rho \times u \times b dy) U^2 - \frac{1}{2} (\rho \times u \times b \times dy) u^2.$

$= \frac{1}{2} [(\rho \times u \times b \times dy) (U^2 - u^2)]$ --- (3).

Total loss of K.E. of fluid passing through BC in fig.

$$= \int_0^{\delta} \frac{1}{2} [(\rho \times u \times b) (U^2 - u^2)] dy.$$

$$= \frac{1}{2} \rho \times b \int_0^{\delta} u (U^2 - u^2) dy \text{ --- } (4).$$

let; δ^{**} = Distance by which the plate is displaced to compensate for the reduction of K.E.

$= \frac{1}{2} (\rho \times b \times \delta^{**} \times U) U^2 = \frac{1}{2} \rho \times b \times \delta^{**} U^3$ --- (5).

(4) = (5) $\Rightarrow \frac{1}{2} \times \rho \times b \int_0^{\delta} u (U^2 - u^2) dy = \frac{1}{2} \rho b \delta^{**} U^3.$

$$\int_0^{\delta} u (U^2 - u^2) dy = \delta^{**} U^3.$$

$$\boxed{\delta^{**} = \frac{u}{U} \int_0^{\delta} \left(1 - \frac{u^2}{U^2}\right) dy.}$$

(1) Find the displacement feplace the momentum thickness & energy thickness for the velocity distribution in the boundary layer is given by $\frac{u}{U} = \frac{y}{\delta^*}$. where u = velocity at a distance y from the plate & $u = U$ at $y = \delta$, where δ = boundary layer thickness. Also calculate the value of $\frac{\delta^*}{\theta}$.

Sol: Given, $\frac{u}{U} = \frac{y}{\delta}$.

i) Displacement thickness.

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy = \int_0^\delta \left(1 - \frac{y}{\delta}\right) dy = \int_0^\delta 1 \cdot dy - \int_0^\delta \frac{y}{\delta} dy.$$

$$= (y)_0^\delta - \frac{1}{\delta}\left(\frac{y^2}{2}\right)_0^\delta = \delta - \frac{1}{\delta}\left(\frac{\delta^2}{2} - 0\right)$$

$$= \delta - \frac{\delta}{2} = \frac{\delta}{2}.$$

$$\therefore \delta^* = \frac{\delta}{2}.$$

ii) $$\theta = \int_0^\delta \frac{u}{U}\left[1 - \frac{u}{U}\right] dy = \int_0^\delta \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) dy = \int_0^\delta \left(\frac{y}{\delta} - \frac{y^2}{\delta^2}\right) dy.$$

$$= \frac{1}{\delta}\left[\int_0^\delta y \cdot dy - \int_0^\delta \frac{y^2}{\delta} dy = \frac{1}{\delta}\left(\frac{y^2}{2}\right)_0^\delta - \frac{1}{\delta^2}\left(\frac{y^3}{3}\right)_0^\delta.\right.$$

$$= \frac{1}{\delta}\left(\frac{\delta^2}{2}\right) - \frac{1}{\delta^2}\left(\frac{\delta^3}{3}\right) = \frac{\delta}{2} - \frac{\delta}{3} = \frac{\delta}{6}.$$

$$\theta = \frac{\delta}{6}.$$

Energy thickness:- $$\delta^{**} = \frac{u}{U}\int_0^\delta \left(1 - \frac{u^2}{U^2}\right) dy = \frac{y}{\delta}\int_0^\delta \left(1 - \frac{y^2}{\delta^2}\right) dy$$

$$= \frac{1}{\delta}\int_0^\delta y\,dy - \frac{1}{\delta^3}\int_0^\delta y^3 dy = \frac{1}{\delta}\left[\frac{y^2}{2}\right]_0^\delta - \frac{1}{\delta^3}\left[\frac{y^4}{4}\right]_0^\delta = \frac{1}{\delta} \cdot \frac{\delta^2}{2} - \frac{1}{\delta^3} \times \frac{\delta^4}{4}.$$

$$\boxed{\delta^{**} = \frac{\delta}{4}}$$

iv) $$\frac{\delta^*}{\theta} = \frac{\frac{\delta}{2}}{\frac{\delta}{6}} = 3.$$

Characteristics of Boundary layer along a thin Plate:-

UNIT-III

Non-uniform flow in open channels:

Gradually varied flow:- In this case of flow the depth of flow increases (or) decreases gradually in the direction of flow, this change from one depth of flow to another occurs gradually in a distance of applicable length.

Rapidly varied flow:- In this case a sudden change of depth occurs at a particular point of a channel and the change from one depth to another takes place in a distance of very short length.

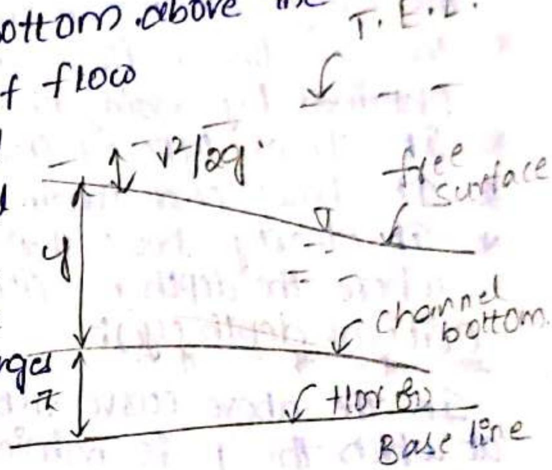
Specific Energy:- It is defined as the energy measured w.r.to the channel bed as datum.

Total Energy $= z + y + \frac{v^2}{2g}$ — per unit width

z = datum (or) Elevation of the channel bottom above the horiz bottom.

y = Depth of flow; v = Avg velocity of flow

The energy per unit width of flowing liquid above the channel bottom, Although the total (or Bernoullis) energy is reduced by friction, the specific energy can increase (or) decrease if bed elevation changes from section to section. however, for uniform flow the specific energy remains constant along the flow.



Specific energy.

If the channel bottom is taken itself as datum the T.E for unit width of liquid.

$$E = y + \frac{v^2}{2g}$$

let $y = P.E = (E_P)$

$$\frac{v^2}{2g} = K.E = (E_K)$$

$$\therefore E = E_P + E_K.$$

let us consider a rectangular section; b = width of channel

y = Depth of flow; Q = Discharge through the channel

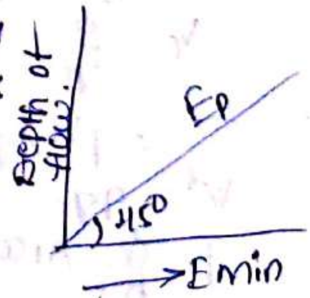
Now velocity of flow;

$$v = \frac{Q}{A} = \frac{Q}{b \times y} = \frac{q \times b}{y \times b} = \frac{q}{y}.$$

q = Discharge per unit width.

$$E = y + \frac{(q/y)^2}{2g} \qquad \boxed{\therefore E = y + \frac{q^2}{2gy^2}}$$

If the above equation is represented graphically then it is known as sp. energy curve. It consists sp. energy against depth of flow.



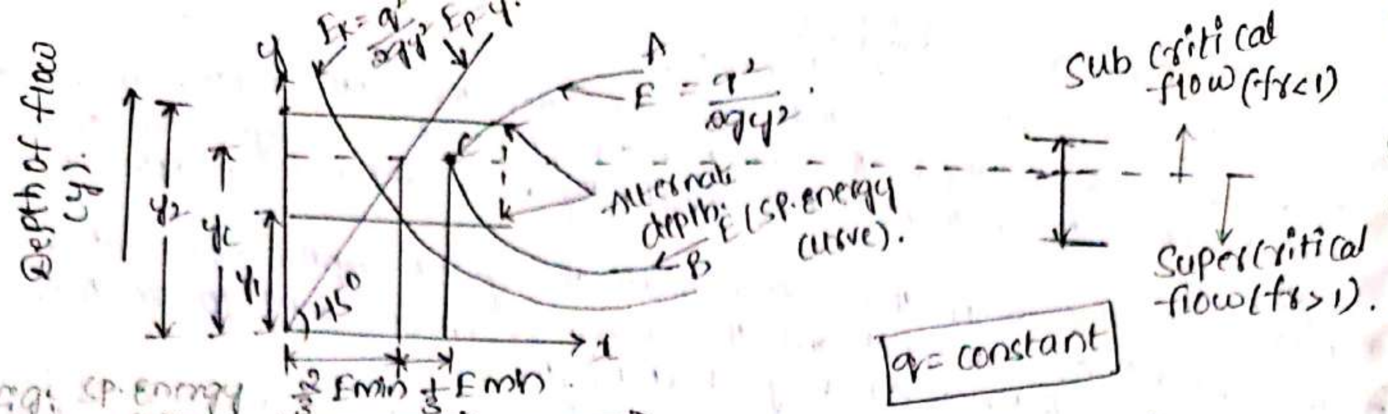


fig: sp. energy curve

* It plots a curve of potential energy with a straight passing through the origin making an angle 45° with x & y axis.
* The curve for kinetic energy is a parabola. plot for sp. energy is obtained by adding kinetic energy to potential energy.
* It shows type of flow i.e. Sub critical, critical & Supercritical.
* It shows corresponding depths.
* It clearly shows that the occurence of minimum sp. energy at where the depth is critical depth.

Critical depth (y_c):-

In the above curve ACB is the critical depth (y_c). The depth of flow at which the 'E' is minimum is called "critical depth(y_c)" critical depth is obtained by differentiate energy equation w.r.t 'y' & equating zero.

$$E = y + \frac{q^2}{2gy^2}$$

$$\frac{dE}{dy} = \frac{d}{dy}\left[y + \frac{q^2}{2gy^2}\right] = 0.$$

$$1 + \frac{q^2}{2g}\left(\frac{-2}{y^3}\right) = 0. \Rightarrow 1 = \frac{2q^2}{2gy^3}.$$

$$y = \left(\frac{q^2}{g}\right)^{1/3}.$$ But when 'E' is min it is critical depth so $\boxed{y_c = \left(\frac{q^2}{g}\right)^{1/3}.}$

Critical velocity (V_c):-

The velocity of flow at critical depth is known as critical velocity (V_c).

$$V = \frac{Q}{A} = \frac{q}{y}. \qquad \left[\because y_c = \left(\frac{q^2}{g}\right)^{1/3}\right].$$

$$V_c = \frac{q}{y_c} = \frac{q}{\left(\frac{q^2}{g}\right)^{1/3}}. \qquad \frac{q}{q^{2/3}} = \frac{q}{q \cdot q^{-1/3}} = q^{1/3}.$$

$$= q^{1/3} \times g^{1/3}.$$

$$V_c^3 = qg.$$

$$q = \frac{Q}{b} \quad \text{Also} \quad q = V_c \times y_c \Rightarrow V_c^3 = V_c \times y_c\, g.$$

$$V_c^3 = V_c \times y_c \times g.$$

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$V_c^2 = y_c \times g$

$$\boxed{V_c = \sqrt{g y_c}}$$

Min sp. Energy in terms of critical depth:

$$E = y + \frac{q^2}{2gy^2}.$$

E is min when depth of flow is critical. Hence $E_{min} = y_c + \frac{q^2}{2gy_c^2}$.

$$\left[\because y_c = \left(\frac{q^2}{g}\right)^{1/3} \text{(or) } y_c^3 = \frac{q^2}{g}\right]$$

$$E_{min} = y_c + \frac{y_c^3 \times g}{2g \times y_c^2}$$

$$= y_c + \frac{y_c}{2}.$$

$$\boxed{E_{min} = \frac{3}{2} y_c} \quad \text{(or)} \quad \boxed{y_c = \frac{2}{3} E_{min}.}$$

Critical flow:- A critical flow is one which E_{min}. A flow corresponding to critical depth is also known as critical flow

$$V_c = \sqrt{g y_c} \qquad \frac{V_c}{\sqrt{g y_c}} = \text{Froude's number}; \ Fr = 1 \rightarrow \text{for critical flow.}$$

$$\boxed{\frac{V_c}{\sqrt{g y_c}} = 1}$$

Sub-critical flow:- The flow is subcritical (or streaming or tranquil) when the depth of flow in a channel is greater than critical depth (y_c).

$*$ $Fr < 1$ $*$ $y > y_c$ $*$ $q < q_c$ $*$ $V < V_c$.

Super-critical flow:- The flow is super critical (or shooting or torrential) when the depth of flow in a channel ~~greater~~ lesser than critical depth (y_c).

$*$ $Fr > 1$ $*$ $V > V_c$ $*$ $y < y_c$ $*$ $q > q_c$.

Condition for maximum discharge for a given value of E.

$$E = y + \frac{V^2}{2g} \quad \text{(or)} \quad E = y + \frac{q^2}{2gy^2}.$$

where; $V = \frac{Q}{A} = \frac{Q}{b \times y}$

$$E = y + \frac{Q^2}{b^2 y^2} \times \frac{1}{2g}.$$

$$E = y + \frac{Q^2}{2gb^2y^2}.$$

$$Q^2 = (2gb^2y^2)(E - y).$$

$$Q = b\sqrt{2g(Ey^2 - y^3)}.$$

$(Ey^2 - y^3)$ should be max.

$$\frac{d}{dy}(Ey^2 - y^3) = 0.$$

$$2Ey - 3y^2 = 0.$$

$$y = \frac{2}{3}E.$$

$$E = \frac{3}{2}y.$$

$$q = \frac{Q}{b} = \frac{10}{8}$$

$$q = 2$$

$$E = y + \frac{q^2}{2gy^2}$$

$$= 2 + \frac{(2)^2}{2 \times 9.81 \times 2^2}$$

$$= 2 + \frac{1}{2 \times 9.81}$$

$$\frac{dE}{dx} = S_0 - S_f$$

$$\frac{dE}{dx} + S_f = S_0$$

$$7.03 \times 10^{-4} + 0.00013 = S_0$$

E' is min when it is equal to $\frac{3}{2}$ times the value of depth of critical flow.
E' is equal to $\frac{3}{2} \times y$ and for max 'Q'. E' value should be at critical depth.

Gradually varied Flow:- If the depth of flow in a channel changes gradually over a long length of the channels, the flow is said to be GVF.

Assumptions:-

1. The bed slope of channel is small.
2. The flow is steady & hence discharge 'Q' is constant.
3. The correction factor (α) is unity (α = KE).
4. The formula such as chezy's & mannings formula which are applicable to the uniform flow are also applicable.
5. The roughness co-efficient is constant.
6. The channel is prismatic.

Derivation:-

Consider a rectangular channel having GVF as shown in fig. The depth of flow is gradually decreasing the direction of flow.

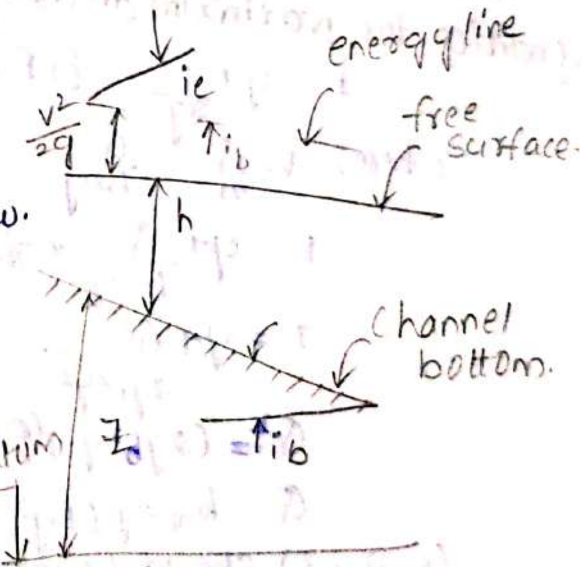


fig: Rectangular channel having GVF.

let

z = height of channel bottom above datum.
h = Depth of flow.
V = mean velocity of flow.
i_b = slope of channel bed
i_e = slope of energy line.
b = width of channel, Q = Discharge through channel.

The energy eqn at any section is given by bernoulli's equation

$$E = z + h + \frac{V^2}{2g} \quad \text{---} (1).$$

Diff. w.r.to 'x'.

$$\frac{dE}{dx} = \frac{dz}{dx} + \frac{dh}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right) \text{---} (2).$$

$\left[\because Q = A \times V \Rightarrow V = \frac{Q}{A} \right.$
$\left. V^2 = \frac{Q^2}{A^2} \right]$

$$\frac{dE}{dx} = \frac{dz}{dx} + \frac{dh}{dx} + \frac{d}{dx}\left[\frac{Q^2}{A^2 2g}\right]. \quad [\because A = b \times h;\ A^2 = b^2h^2].$$

$$\frac{dE}{dx} = \frac{dz}{dx} + \frac{dh}{dx} + \frac{d}{dx}\left[\frac{Q^2}{b^2h^2 2g}\right].$$

$$\frac{dE}{dx} = \frac{dz}{dx} + \frac{dh}{dx} + \frac{Q^2}{b^2 2g} \times \frac{d}{dx}\left[\frac{1}{h^2}\right].$$

$$\frac{dE}{dx} = \frac{dz}{dx} + \frac{dh}{dx} + \frac{Q^2}{b^2 2g}\left[\frac{-2}{h^3}\right]\frac{dh}{dx}.$$

$$\frac{dE}{dx} = \frac{dz}{dx} + \frac{dh}{dx} - \frac{Q^2}{A^2 gh} \times \frac{dh}{dx}.$$

$$\frac{dE}{dx} = \frac{dz}{dx} + \frac{dh}{dx} - \frac{V^2}{gh} \times \frac{dh}{dx}$$

$$\frac{dE}{dx} = \frac{dz}{dx} + \frac{dh}{dx}\left[1 - \frac{V^2}{gh}\right] \text{ --- } (3).$$

Now;

$\frac{dE}{dx}$ = slope of energy line $= -i_e$; $\frac{dz}{dx}$ = slope of channel bed $= -i_b$.

(-ve) of i_e & i_b is taken as with the increase in the value of 'x' value of 'E' and 'z' decreases. Sub $\frac{dE}{dx}$ & $\frac{dz}{dx}$ in eqn (3).

$$-i_e = -i_b + \frac{dh}{dx}\left[1 - \frac{V^2}{2gh}\right].$$

$$\boxed{\frac{dh}{dx} = \frac{i_b - i_e}{\left[1 - \frac{V^2}{2gh}\right]}} \quad (or) \quad \boxed{\frac{dh}{dx} = \frac{i_b - i_e}{1 - F_e^2}}$$

where;

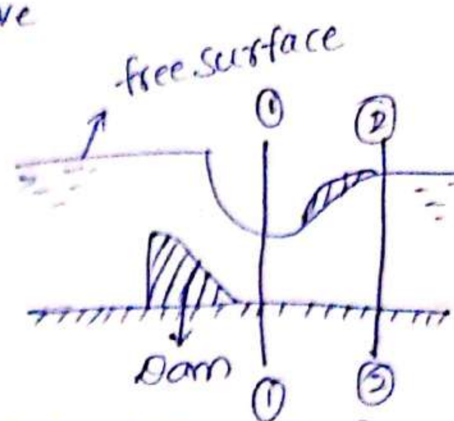
h = Depth of flow; $\frac{dh}{dx}$ = Variation of depth of flow along length.

Hydraulic Jump:- The rise of water level, which takes place due to the transformation of the flow unstable shooting flow to a stable streaming flow. Loss of energy due to eddy formation and turbulence occurs.

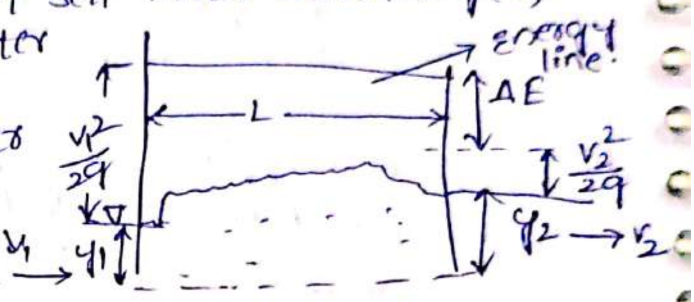
* It is known as standing wave, because it is essence, a wave which is stationary (or) stand still at one place. This phenomenon occurs when:
* A super-critical flow stream tries to reach it is alternate depth in sub-critical mode.
* A flow whose depth is less than critical depth meets another flow with depth more than critical depth.
* When doing so it generates considerable disturbances in the form of large-scale eddies and a reverse flow roller with the result of that the jump falls short of its alternate depth.
* In this process it loses substantial energy.
* The depths on either side of jump are known as sequent depth (or) conjugate depths.

Hydraulic Jump (or) standing wave:

consider the flow of water over a dam as shown in fig. The height of water at Section ① - ① is small. As we move towards d/s, the height & depth of water increases rapidly over a short length of the channel. This is because at section ① - ① flow is super-critical/shooting flow/Torrential flow as the depth of water at section ① - ① is less than critical depth.

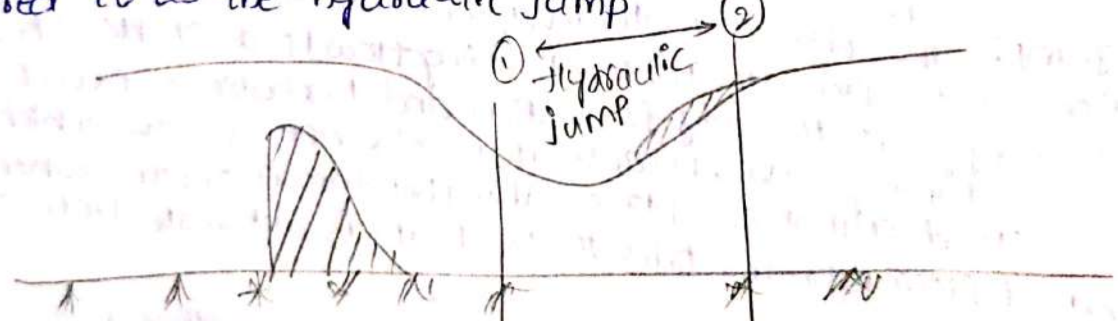


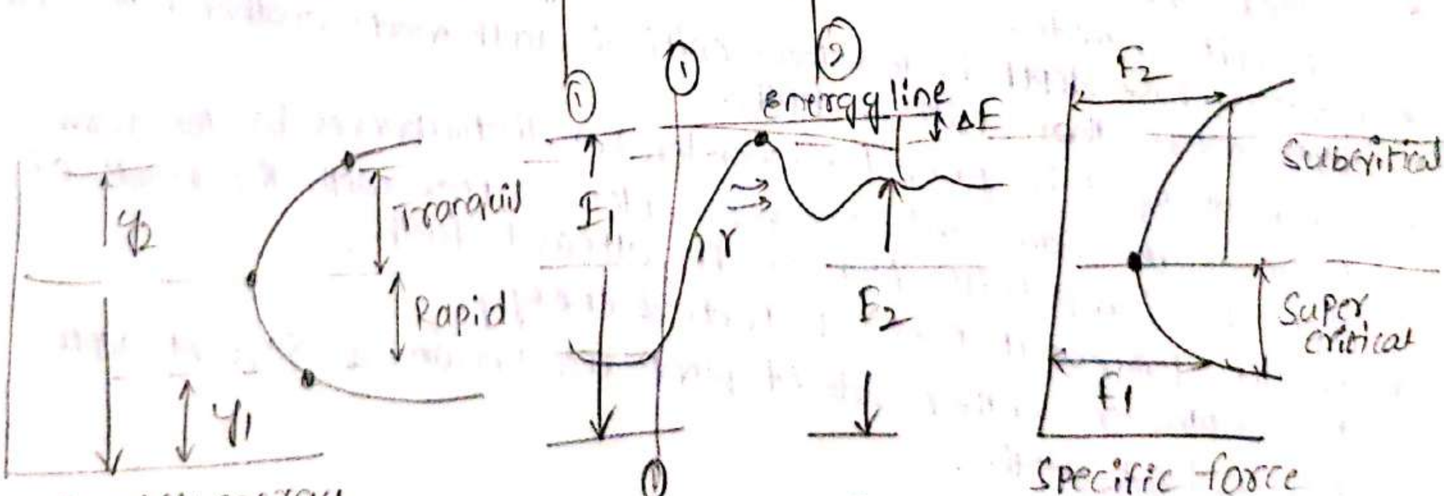
* Super critical flow is the unstable flow and does not continue on the d/s side.
* Then this shooting flow will convert it self into a streaming (or) tranquil flow and hence depth of water will increase.
* This sudden increase of depth of water is called a Hydraulic jump (or) standing wave.



Hydraulic Jump:-

The hydraulic jump is defined as the sudden and turbulent passage of water from a super critical state off to sub-critical depth state. It has been classified as rapidly varied flow i.e the change in depth of flow from rapid to transquil state for a short distance. This phenomenon of sudden increase in depth of flow is reffered to as the hydraulic jump.





Specific energy curve.

Hydraulic Jump.

Specific force curve.

$\boxed{\Delta E = E_1 - E_2}$

The flow in hydraulic jump is considered accompained by the formation of external turbulent rollers & there is a considerable dissipation of energy.

Assumptions for analysis of hydraulic jump:

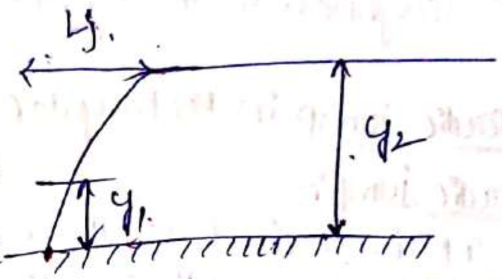
The following assumptions are related to the analysis of H.J.

* It is assumed that before & after jump formation the flow is uniform and the pressure distribution is hydrostatic (or) same pressure
* The length of jump is small so that the losses due to friction on the channel floor are small and hence neglected.
* The channel flow is horizontal (or) shape that the weight components of water mass comprising the jump is neglegible small.

Some properties of Hydraulic Jump:

1. Length of the Jump. 3. water surface profile.
2. pressure Distribution. 4. velocity profile.
5. other characteristics.

1. Length of Jump (L_J):



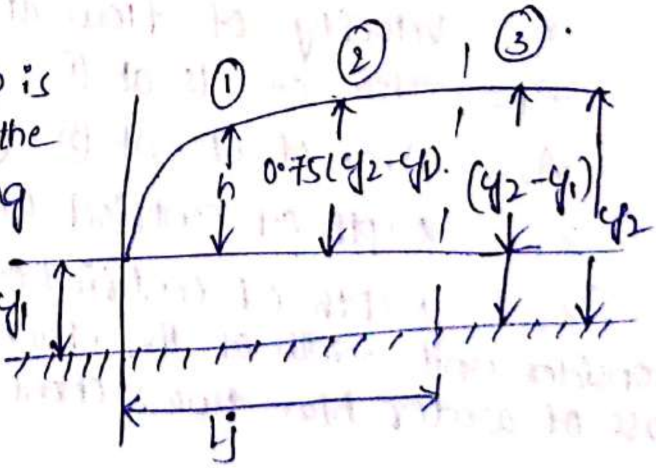
The length of jump Lj is an important parameter affecting the size of the stilling basin in which the Jump is used. It is usually to take the length of jump as the horizontal distance b/w toe of the jump to the section where the water surface reaching the maximum depth Lj can be expressed as

$L_j = 6.9(y_2 - y_1)$.

2. Pressure Distribution:

The pressure distribution of the toe (starting state) of the jump & at the end of the jump follow some pressure distribution. At the inside of the body of toe jump the strong curvature of the stream lines curve the pressure to derivate from the hydrostatic (or) same distribution. The defect from the hydrostatic pressure increase with increase in the intial froude number F_r.

3. water surface profile (n):



The surface profile of the jump is useful in the efficient design of the side walls & the fitter of stilling basin. Assuming the co-ordinates of the profile are (x, h) with boundary condition that at $x=0$, $h=0$, $x = L_j$, $h = y_2 - y_1$. In general $h = f(x, F_1)$.

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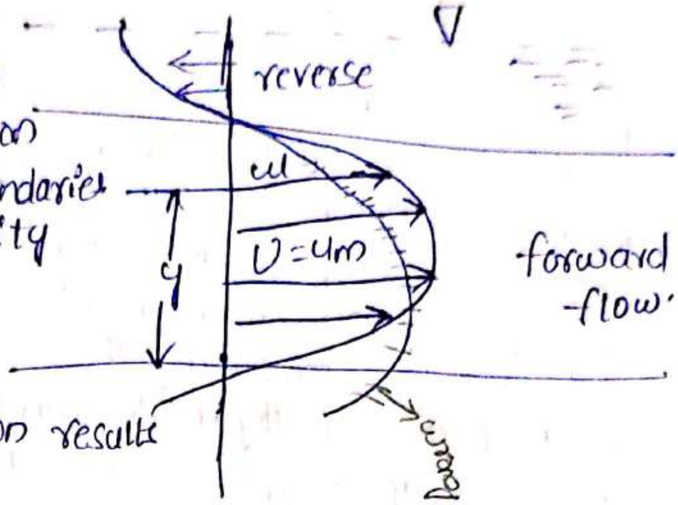
Based on analysis of a large no. of jump profile & bed slopes & it is expressed as $\boxed{\eta = \frac{h}{0.75(y_2 - y_1)}}$ - from graph.

velocity profile:-



when the subcritical stream entre the jump body from toe (or) lower and undergoes shearing action at the top as well as the solid boundaries The top surface has high velocity with respect to the fluid mass.

For every excess forwarded fluid flow a reverse flow exists at the top surface. This situation results in the formation of roller.

Other characteristics:-

It has been found that the initial boundary layer thickness & relative roughness of the bed play a major role in for hydraulic jump.

Hydraulic jump in Rectangular channel (or) Horizontal channel:-

Hydraulic jump:-

It is defined as the rapid variation of flow from a super critical state. The depth of flow before the jump is known as initial depth & it is denoted by (y_1) & the depth of flow after the jump is as sequent depth (y_2).

consider hydraulic jump formed in a channel of horizontal bed consider two sections ① - ① & ② - ② before & after hydraulic jump.

let; y_1 = Depth of flow at section ① - ①.

y_2 = Depth of flow at section ② - ②.

v_1 = Velocity of flow at section ① - ①.

v_2 = Velocity of flow at section ② - ②.

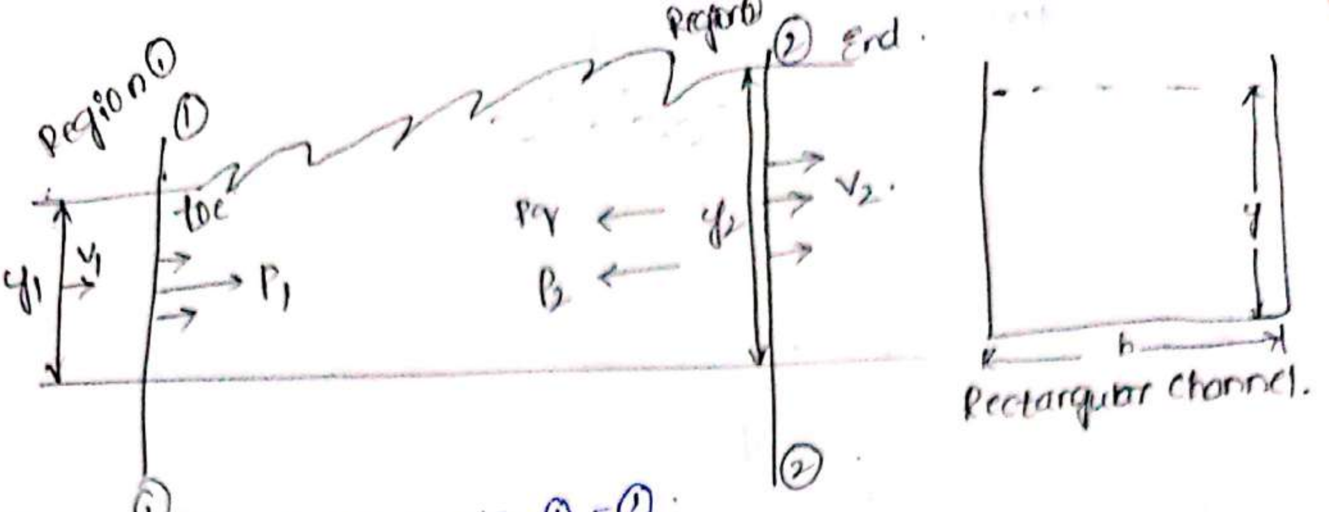
A_1 = Area of C/s at ① - ①.

A_2 = Area of C/s at ② - ②.

$\bar{Z}_1$ = Depth of centriod of area @ ① - ①.

$\bar{Z}_2$ = Depth of centriod of area @ ② - ②.

Consider unit width of the channel, the forces acting on the mass of water b/w two sections ① - ① & ② - ② are



① Pressure force P_1 at section ① - ①.
② Pressure force P_2 at section ② - ②.
③ Frictional force on the floor of the channel which assumed to the neglected. Hence in accordance with the momentum equation.

$F(dt)$ = mass change in velocity.

(force in pressure force).

$(P_2 - P_1) = \rho Q (V_1 - V_2)$ — ①

ρQ = mass of fluid

$(V_1 - V_2)$ = (Initial and Final velocities)

In case of hydrostatic pressure distribution the pressure force at any section. i.e. $P = wA\bar{z}$ — ② [∵ w = SP. wt of water] $\left(w = \rho g,\ \rho = \frac{w}{g}\right)$

Sub ② in ①

$(wA_2\bar{z}_2 - wA_1\bar{z}_1) = \frac{w}{g} Q\left(\frac{Q}{A_1} - \frac{Q}{A_2}\right)$

$$\frac{Q^2}{gA_1} + A_1\bar{z}_1 = \frac{Q^2}{gA_2} + A_2\bar{z}_2 \quad — ③$$

Area of rectangle channel is $A_1 = B \times D$ for unit width $B = 1$

$w = \rho g;\ \rho = \frac{w}{g}$

$$\left.\begin{aligned} A_1 &= B \times y_1 \\ A_2 &= B \times y_2 \end{aligned}\right\} — ④$$

$\bar{z}_1 = \left(\frac{y_1}{2}\right);\ \bar{z}_2 = \left(\frac{y_2}{2}\right)$.

Sub $A_1, A_2, \bar{z}_1$ & $\bar{z}_2$ in eqn ④.

$$\frac{Q^2}{gy_1} + y_1 \times \frac{y_1}{2} = \frac{Q^2}{gy_2} + y_2 \times \frac{y_2}{2}.$$

$$\frac{Q^2}{gy_1} + \frac{y_1^2}{2} = \frac{Q^2}{gy_2} + \frac{y_2^2}{2}.$$

$$\frac{Q^2}{g}\left[\frac{1}{y_1} - \frac{1}{y_2}\right] = \frac{1}{2}\left[y_2^2 - y_1^2\right].$$

for unit width $q = \frac{Q}{B}, = \frac{Q}{1} \Rightarrow Q = q$

$$\frac{q^2}{g}\left[\frac{1}{y_1} - \frac{1}{y_2}\right] = \frac{1}{2}\left[y_2^2 - y_1^2\right].$$

$$\frac{2q^2}{g}\left[\frac{y_2 - y_1}{y_1 \cdot y_2}\right] = (y_2 - y_1)(y_1 + y_2)$$

$$\boxed{\frac{2q^2}{g} = y_1 \times y_2 (y_1 + y_2)} \quad — ⑤$$

→ Relation b/w q & sequent depths

Eqn ⑤ is the momentum equation for hydraulic jump in rectangular channel.

$$\frac{2q^2}{gy_1} = y_2 \times y_1 + y_2^2$$

$$\boxed{y_2^2 + y_1 y_2 - \frac{2q^2}{gy_1} = 0} \quad — ⑥.$$

Eqn (6) is a quadratic equation in y_2 & hence the solution is

$$Q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2\times a}$$

$$y_2 = \frac{-y_1 \pm \sqrt{y_1^2 - \frac{4 \cdot 8 q^2}{g y_1}}}{2\times 1} \Rightarrow y_2 = -y_1 \pm \sqrt{\frac{y_1^2}{4} + \frac{2q^2}{g y_1}}.$$

The two roots of equation are

$$-\frac{y_1}{2} - \sqrt{\frac{y_1^2}{4} + \frac{2q^2}{g y_1}} \quad \& \quad -\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{2q^2}{g y_1}}$$

Negative root is negligable & conside (+ve) root.

$$y_2 = -\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{2q^2}{g y_1}} \quad \text{— (6)-i.}$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + \frac{8q^2}{g y_1^3}}\right]$$

Since for a rectangular channel $\frac{q^2}{g} = y_c^3$, where y_c = critical depth.

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8\left(\frac{y_c}{y_1}\right)^3}\right] \quad \text{— (7).}$$

This is the expression for hydraulic jump in rectangular channel.

This is the expression for hydraulic jump in terms of froude number.

Eqn (7) can be also expressed in terms of froude number.

$$F_r = \frac{V}{\sqrt{gy}} \; ; \; q = \frac{Q}{B} = \frac{A \times velocity}{B} = \frac{B \times y \times V}{B} \; ; \; V = \frac{q}{y}.$$

$$F_r = \frac{q}{\sqrt{y^3 g}}.$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1 + 8(F_r)^2} - 1\right]$$

$$\boxed{y_2 = \frac{y_1}{2}\left[\sqrt{1 + 8(F_r)^2} - 1\right]}.$$

The above eqn belongs depth in terms of froude number.

Expression for loss of energy due to hydraulic jump:-

The energy loss in hydraulic jump i.e. due to formation of eddies and turbulence occurs. This loss of energy ΔE is computed at two sections as $h_j = E_1 - E_2$

$$\Delta E = E_1 - E_2 \quad \text{— (1)}$$

$$= \left[y_1 + \frac{V_1^2}{2g}\right] - \left[y_2 + \frac{V_2^2}{2g}\right] \qquad [B_1 = B_2 = 1] \text{ unit width.}$$

$$Q = A_1 V_1 = A_2 V_2$$

$$= B_1 V_1 y_1 = B_2 y_2 V_2.$$

$$V_1 = \frac{Q}{B_1 y_1} = \frac{Q}{y_1} \text{ (8)} \quad V_2 = \frac{Q}{B_2 y_2} = \frac{Q}{y_2}.$$

$$\Delta E = \left[y_1 + \frac{Q^2}{2g(y_1)^2}\right] - \left[y_2 + \frac{Q^2}{2g(y_2)^2}\right].$$

$$\frac{Q^2}{2gy_1^2} - \frac{Q^2}{2gy_2^2} - (y_2 - y_1).$$

$$\left[\frac{q^2}{2gy_1^2} - \frac{q^2}{2gy_2^2}\right] - [y_2 - y_1]$$

$$\frac{q^2}{2g}\left[\frac{1}{y_1^2} - \frac{1}{y_2^2}\right] - y_2 - y_1$$

$$\Delta E \Rightarrow \frac{q^2}{2g} \frac{(y_2^2 - y_1^2)}{(y_1 \times y_2)^2} - (y_2 - y_1)$$

Sub $\left[\frac{q^2}{g}\right] = \frac{1}{2} y_1 \times y_2 \, [y_1 + y_2]$ [From previous derivation from eqn (5)]

$$\Delta E = \frac{\frac{1}{4}(y_1 \times y_2)(y_1 + y_2)(y_2^2 - y_1^2) - (y_2 - y_1)}{(y_1 \times y_2)^2}.$$

on simplification we get $\Delta E = \frac{(y_2 - y_1)^3}{4y_1 y_2}$ —— (2).

eqn (2) is expressed in terms of velocity $\Delta E = \frac{(V_1 - V_2)^2}{2g(V_1 + V_2)}$.

where V_1 & V_2 are the mean velocities of flow after and before jump.

Height of Jump (hj):-

The height of the jump may be defined as the difference b/w the depths of after & before the jump $h_j = \frac{y_1}{2}\left[\sqrt{1 + 8Fr^2} - 1\right] - y_1 = \frac{y_1}{2}\left[\sqrt{1+8Fr^2} - \frac{y_1}{2} - y_1\right]$

$$h_j = \frac{-3y_1}{2} + \frac{y_1}{2}\sqrt{1 + 8Fr^2}. \qquad \boxed{\therefore h_j = (y_2 - y_1)}.$$

Length of Jump (Lj):-

It may be defined as the distance measured from the front face of the jump, to a point on the surface, immediately down stream from the roller for a rectangular channel the length of the jump Lj varies b/w 5 to 7 times the height of jump.

$$L_j = (5 \text{ to } 7)\, h_j.$$

$$L_j = (5 \text{ to } 7)(y_2 - y_1).$$

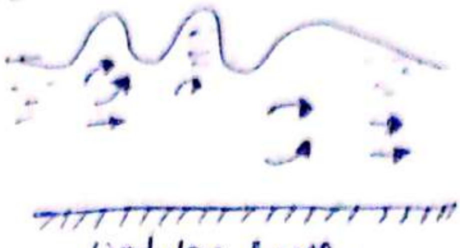
Types of hydraulic jump:-

They are five different types of hydraulic jump which may occur on a horizontal floor they are:-

1. undular jump.
2. weak jump.
3. oscillating jump.
4. steady jump.
5. strong jump.

1. Undular Jump:

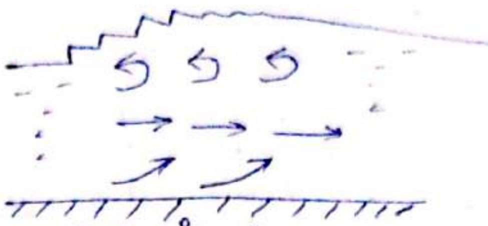
For Fr = 1.0 to 1.7 the water surface undulations and the jump is called an undular jump. These waves are small and gradually diminish in amplitude.



Undular jump.

Weak Jump:

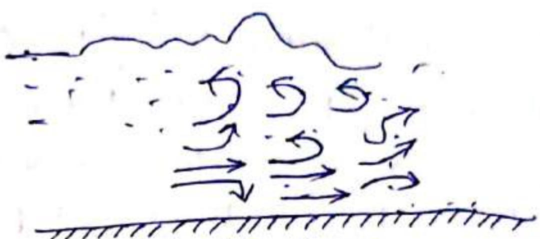
For Fr_1 = 1.7 to 2.5 the jump formed is called weak jump as the velocity throw out is fairly uniform and only a small amount of energy of is dissipated.



weak jump

Oscillating Jump:-

For Fr_1 = 2.5 to 4.5 jump formed is known as an oscillating jump. In this case the entering jet of water oscillating back & forth from bottom to the surface & back against formation of large waves of irregular period.



oscillating jump.

Steady Jump:

For Fr_1 = 4.5 to 9.0 the jump formed is well stabilised & is called a steady jump for thus jump the energy dissipation ranges from 45 to 75%.

steady jump.

Strong Jump:-

For fr > 9.0 & larger the jump formed called a strong jump and energy dissipation ranges may reach 85%.

Strong jump

Applications of hydraulic jump:- *

* It raises water level in the channels for irrigation etc...
* It increases the discharge through a sluice by a holding back the tailwt.
* It may be used for mixing chemicals in water & other liquids.
* To operate flow measurement flumes efficiently.
* It acts as energy dissipation to dissipate the excess energy of water flowing from down stream of spillways, sluice states etc...
* In desalination of sea water.

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Location of hydraulic jump:

A hydraulic jump is formed whenever the momentum equation is satisfied b/w the super critical and sub critical parts of a stream in connection which G.V.F calculation. It has already been indicated that the control for super critical flow is at upstream end & for sub critical flows the control is at the down stream end. The location of hydraulic jump satisfy these requirements.

Characteristics of hydraulic jump:

1. Relative loss:

It is defined as the ratio of loss of energy to the specific energy before the jump. Relative loss $= \frac{\Delta E}{E_1}$.

2. Efficiency of the jump:

It is defined as the ratio of specific energy after the jump to the specific energy before jump. ∴ Efficiency of Jump $= \frac{E_2}{E_1}$.

3. Relative height of Jump:

It is defined as the ratio of height of the jump to the specific energy before the jump. ∴ Relative height $= \frac{d_2-d_1}{d_1}$.

① The depth of flow of water at a certain section of a rectangular channel of 4m wide is 0.5m. This discharge through the channel is $16 m^3/s$. If a hydraulic jump takes place on the down stream side. Find the depth of flow after the jump.

Sol: Given,

$b = 4m$

Depth of flow of water before jump $d_1 = 0.5m$.

Discharge $Q = 16 m^3/sec$.

Discharge per unit width $q = \frac{Q}{b} = \frac{16}{4} = 4 m^2/sec$.

Let depth of flow after jump $= d_2$.

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}} = -\frac{0.5}{2} + \sqrt{\frac{(0.5)^2}{4} + \frac{2(4)^2}{9.81 \times 0.5}}.$$

$$d_2 = -0.25 + \sqrt{0.0625 + 6.5239}.$$

$$\therefore d_2 = 2.316m.$$

② The depth of flow of water, at a ~~circular~~ certain section of a rectangular channel of 2m wide is 0.3m. The discharge through the channel is $1.5 m^3/sec$. Determine whether a hydraulic Jump will occur and if so. Find its height and loss of energy per kg of water.

Sol: Given,

$d_1 = 0.3m$

$Q = 1.5 m^3/sec$

$b = 2m$.

$q = \frac{Q}{b} = \frac{1.5}{2} = 0.75 m^3/s$.

Hydraulic jump will occur if the depth of flow on the upstream side is less than the critical depth on upstream side (or) if the froude number on u/s side is more than one.

Critical depth $(h_c) = \left[\frac{q^2}{g}\right]^{1/3} = \left[\frac{(0.75)^2}{9.81}\right]^{1/3}$ $\therefore h_c = 0.3859$.

The depth on u/s side is 0.3. Depth on u/s side is less than the critical depth on u/s. Hence hydraulic jump will occur.

Depth of flow after hydraulic jump

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}} = -\frac{0.3}{2} + \sqrt{\frac{(0.3)^2}{4} + \frac{2(0.75)^2}{9.81\times0.3}}$$

$\therefore d_2 = 0.4862\,m$.

Height of hydraulic jump $= d_2 - d_1 = 0.4862 - 0.3 = 0.1862\,m$.

Loss of energy per kg of water due to hydraulic jump.

$$h_L = \frac{(d_2-d_1)^3}{4d_1d_2} = \frac{[0.4862 - 0.30]^2}{4\times0.4862\times0.30}$$

$\therefore h_L = 0.01106\,m\text{-}kg/kg$.

(3) A sluice gate discharges water into a horizontal rectangular channel with a velocity of 10m/s and depth of flow of 1m. Determine the depth after the jump and consequent loss in total head.

Sol: Given,

Velocity of flow befor hydraulic jump $V_1 = 10\,m/s$.

Depth of flow before hydraulic jump $d_1 = 1m$.

Discharge per unit width $q = V_1 \times d_1 = 10\times1 = 10\,m^2/sec$.

Depth of flow after jump $d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}} = -\frac{10}{2} + \sqrt{\frac{1^2}{4} + \frac{2\times10^2}{9.81\times1}}$

$= 4.043\,m$.

Loss in total head $h_L = \frac{(d_2-d_1)^3}{4d_1d_2} = \frac{(4.043-1.0)^3}{4\times1.0\times4.043}$

$h_L = 1.742\,m$.

(4) A sluice gate discharges water into a horizontal rectangular channel with a velocity of 6m/s & depth of flow is 0.4m. The width of the channel is 8m. Determine whether a hydraulic jump will occur and if so, find it's height and loss of energy per kg of water. Also determine the power lost in the hydraulic jump.

Sol: Given,

1) $V_1 = 6\,m/s$; $d_1 = 0.4\,m$; $b = 8\,m$

Discharge per unit width $q = \frac{Q}{b} = \frac{V_1\times A}{b}$

$= \frac{V_1\times d_1\times b}{b}$

$q = V_1 d_1 = 6\times0.4 = 2.4\,m^3/sec$

Froude number on u/s side $(F_e) = \frac{V_1}{\sqrt{gd_1}} = \frac{6.0}{\sqrt{9.81\times0.4}} = 3.0287$

As the froude number is more than one, the flow is shooting on the u/s side. Shooting flow is unstable flow and it will convert itself into streaming flow by raising its height and hence hydraulic jump will take place.

ii) let the depth of hydraulic jump = d_2

$$d_2 = \frac{d_1}{2}\left[\sqrt{1+8(F_e)^2}-1\right] = \frac{0.4}{2}\left[\sqrt{1+8(3.029)^2}-1\right] \Rightarrow d_2 = 1.525m.$$

$\therefore$ Height of hydraulic jump $= d_2 - d_1 = 1.525 - 0.4 = 1.125m$

iii) loss of energy per kg of water is given by $h_L = \frac{(d_2-d_1)^3}{4d_1d_2}$

$$= \frac{(1.525-0.4)^3}{4\times0.4\times1.525}.$$

$h_L = 0.5835$ m-kg/kg of water.

iv) power lost in KW $= \frac{\rho g \times Q \times h_L}{1000}$

$Q = V\times A \Rightarrow V_1 d_1 b$

$= 6\times0.48\times8$

$= 1.92 m^3/s.$

$$P = \frac{1000\times9.81\times1.92\times0.5835}{1000} = 109.9KW.$$

⑤ A hydraulic jump forms at the down stream end of spill way carrying $17.93\ m^3/s$ discharge. If the depth before jump is 0.80m determine the depth after the jump and energy loss.

Sol: Given,

Discharge $(Q) = 17.93 = m^3/s.$

Depth of flow before jump $= 0.80m = d_1$

let width $= b = 1m$; then $q = \frac{Q}{b} = \frac{17.93}{1}\ m^2/sec$.

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}}$$

$$= -\frac{0.8}{2} + \sqrt{\frac{(0.8)^2}{4} + \frac{2\times17.93^2}{9.81\times0.8}}.$$

$d_2 = 8.66m.$

Loss of energy $h_L = \frac{(d_2-d_1)^3}{4d_1d_2}$

$$= \frac{(8.66-0.8)^3}{4\times0.8\times8.66}.$$

$= 17.52m.$

Characteristics of hydraulic Jump:

Basic characteristics of hydraulic jump are

1. Conjugate depths.
2. energy loss in the jump.
3. Height of the jump.
4. length of the jump.

1. Conjugate depths / Alternate / Sequent depths :-

The conjugate depths of the hydraulic jump are y_1 and y_2. are known as conjugate depths because these are the depths above and below the critical depth.

$$\boxed{\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1+8(Fr)^2} - 1\right]}$$

$Fr_1 = \frac{V_1}{\sqrt{gh_1}}$ is a froude number; y_1 = Depth of Super-critical flow.

y_2 = Depth of Sub-critical flow; V_1 = velocity of Super-critical flow.

g = Acceleration due to gravity.

2. Energy loss in the jump:-

Loss of energy in the jump is equal to the difference in specific energy before and after the jump. $\boxed{\Delta E = E_1 - E_2 = \frac{(y_2-y_1)^3}{4y_1y_2}}$.

E_1 E_2 are the specific energy before and after.

E_1 = Sp. Energy of Super critical flow, y_1 = Depth of Super-c flow

E_2 = Sp. Energy of sub critical flow, y_2 = Depth of sub-critical flow

3. Height of the jump (H_j):-

Height of the jump is equal to the difference between the depths before and after the jump. $\boxed{h_j = h_2 - h_1}$

4. Length of the jump (L_j):-

Length of the hydraulic jump may be defined as the distance measured from the front face of the jump to a point on the surface immediately downstream from the roller.

$20 = 0.5\sqrt{8fr^2}$

$\frac{20}{0.5}$

$40 = \sqrt{8(fr)^2}$

$40 = \sqrt{8}\cdot\sqrt{fr^2}$

$fr = \frac{40}{\sqrt{8}}$

$6.4^3 = 8fe^2 + 1$

$5.403 = 8fe^2$

$fe^2 = \frac{5.403}{8}$

$fe = 0.625$

$\frac{y_2}{y_1} = 0.5\left\{\sqrt{8fe^2+1} - 1\right\}$

$20 = 0.5\left\{\sqrt{8fe^2+1}\right\} - 1$

$40 = \left(\sqrt{8fe^2+1}\right)$

$41 = \sqrt{8fe^2+1}$

UNIT-V — Pumps
Centrifugal pumps

Introduction:-

Hydraulic machines which convert the mechanical energy into hydraulic energy are called "pumps". Hydraulic energy is in the form of pressure energy.

A pump is defined as a mechanical device which when inserted in a pipe line converts the mechanical energy supplied to it from some external source into hydraulic energy and transfers the same to the liquid through the pipe line, there by increasing the energy of flowing liquid which is subsequently converted into potential energy as the liquid is lifted from a lower to higher level.

Pumps need a prime mover to run and get mechanical energy which gets transformed into hydraulic energy pumps raise liquids to a higher level from a lower level by creating a low pressure at inlet and high pressure at their out let. Due to low inlet pressure, the liquid rise from where it is available and the high outlet pressure forces it up where it is to be stored & supplied.

Classification of pumps:- various types of pumps may be broadly classified as 1. positive displacement pumps. 2. Roto dynamic pumps.

Positive displacement pumps:-

In this type, liquid is sucked and then it is actually pushed (or) displaced due to the thrust exerted on it by a moving member (piston), which results in lifting the liquid to the required height.

Eg:- Reciprocating pump.

Roto dynamic pumps:-

Roto dynamic pumps have a rotating element called impeller, through which as the fluid passes its angular momentum changes, due to which the pressure energy of the liquid is increased. The pressure thus increased lifts the liquid from lower to higher level.

Eg:- centrifugal pumps.

Component parts of a centrifugal pumps:-

1. Impeller 2. casing 3. suction pipe 4. Delivery pipe.

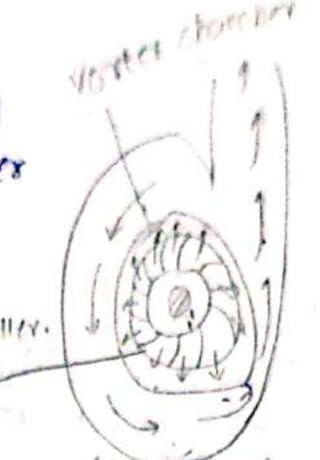
Impeller:- An impeller is a wheel with a series of backward curved vanes. It is mounted on a shaft which is usually coupled to an electric motor. The pressure thus increased, lifts the liquid from lower to higher level.

Casing:- The casing is air tight chamber surrounding the pump impeller. It contain suction and discharge arrangements, supporting for bearings and facilites to house the rotor assembly.

a. Volute casing: In this type of casing the area flow gradually increases from the impeller outlet to the delivery pipe so as to reduce the velocity of flow.

b. Vortex casing:

If a circular chamber is provided between the impeller and the volute chamber the casing is known as vortex casing. By introducing the circular chamber loss of energy due to formation of eddies is reduced and hence efficiency is increased.



Vortex casing.

c. Casing with Guide Blades:-

The casing in which the impeller is surrounded by a series of guide blades mounted on a right which is known as diffuser. Guide vanes are designed in such a way that the water from the impeller enters the guide vanes without shocks. Area of guide vane increases, thus reducing the velocity of flow through guide vanes and consequently increasing the pressure of water.

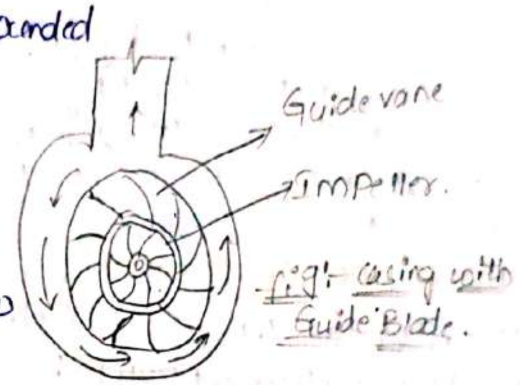


fig: casing with Guide Blade.

water from the guide vane passes into the surroundings casing which may be circular and concentric with impeller.

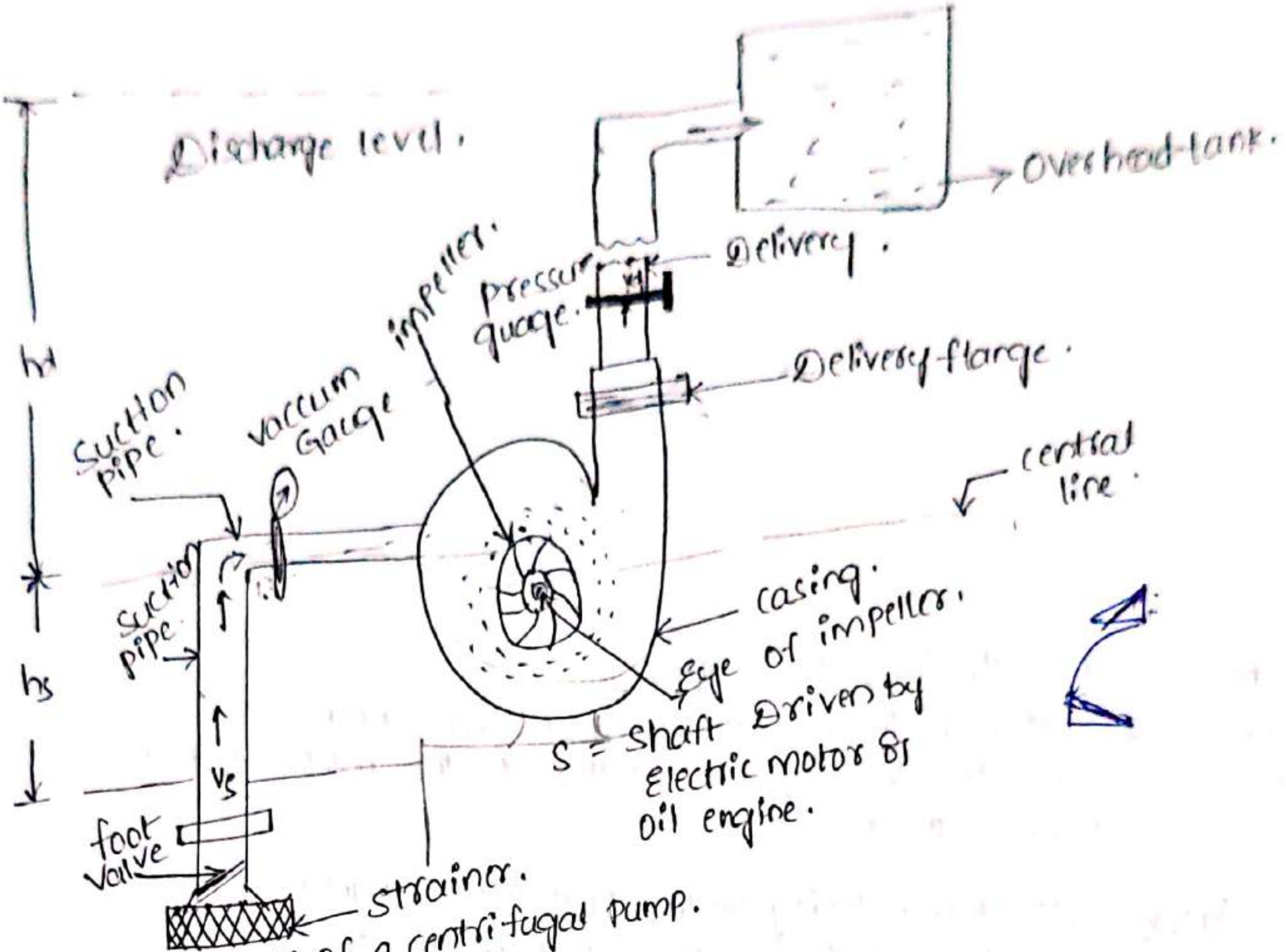
Suction pipe with a foot valve and a strainer:-

A pipe whose upper end is connected to the invert of the pump (eye of the impeller) and its lower end dips into the sump from which the liquid is to be lifted, is known as a "suction pipe". The pipe is laid air-tight so that there is no possibility of formation of air pockets.

Suction pipe is provided with a strainer as its lower end so as to prevent the entry of solid particles debris etc - into the pump. A foot valve which is non-return & one-way valve is fitted above the strainer into the suction pipe. Foot valve serves to fill the pump with liquid before it is started and prevents back-flow when the pump stopped.

Delivery pipe:-

A pipe whose one end is connected to the outlet of the pump and other end delivers the water to the required height is known as delivery pipe. A regulating valve is provided the delivery pipe to regulate the supply of water.



Main parts of a centrifugal pump.

working of centrifugal pump:-

The working of a centrifugal pumps strats with priming is the process of filling the suction pipe, casing and the delivery pipe upto the delivery valve by the liquid which is to be pumped. Priming is done to drive out the air pockets present.

* once priming is done, keeping the delivery valve still closed the electric motor is started which rotates the impeller. The centrifugal force induced due to forced vortex increases the pressure energy of the liquid. As long as the delivery valve is closed, the liquid gets churned inside the casing and gets more energy.

once the delivery valve is opened, the liquid rushes into the delivery pipe. This empties the casing of the centrifugal pump creating a partial vaccum at the centre of the pump, there by the making the continuous process of pumping of the liquid.

workdone by the centrifugal pump:-

Incase of the centrifugal pump work is done by the impeller on the water. The expression for the working workdone by the impeller on the water is obtained by drawing velocity triangles at inlet and outlet of the impeller in the impeller in the same way as explained for turbines.

The water enters the impeller at its centre (radially i.e $\alpha = 90^\circ$ and $V_{w1} = 0$) and leaves at its outer periphery for drawing the velocity triangles, the same notations are used as that wave earlier.

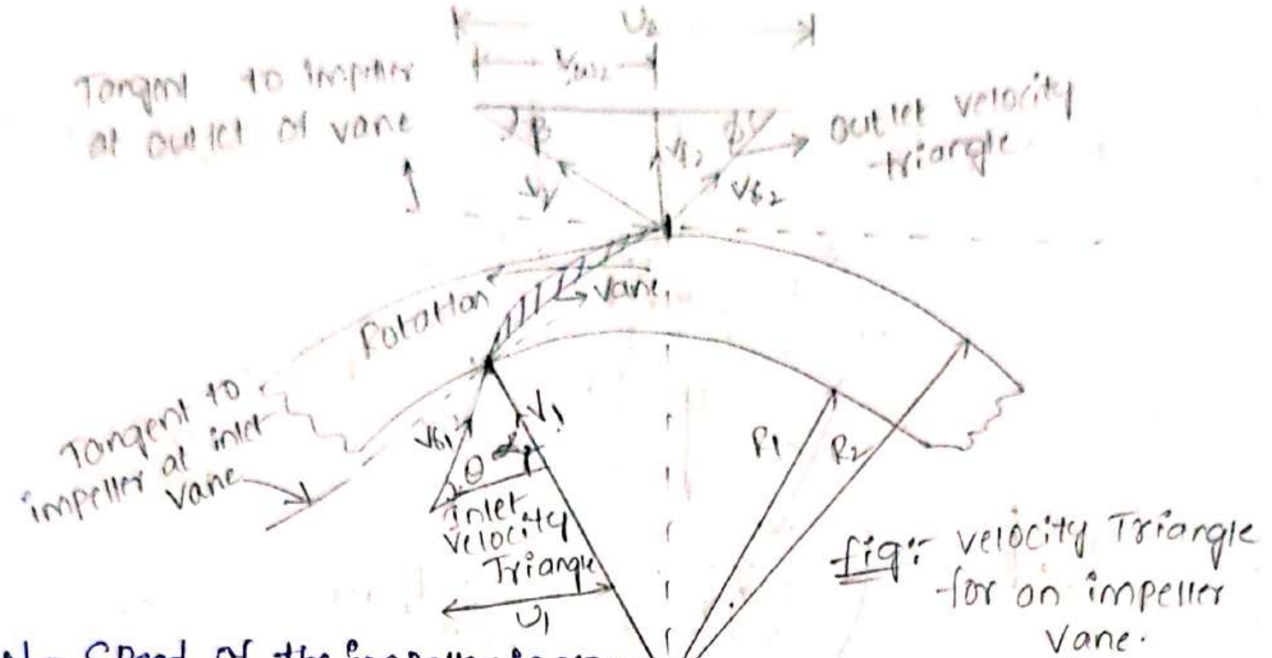


fig: velocity Triangle for an impeller vane.

N = Speed of the impeller in rpm.

D_1 & D_2 = Diameter of the impeller at inlet and outlet.

U_1 & U_2 = Tangential velocity of impeller at the inlet & outlet.

$= \frac{\pi D_1 N}{60}$ & $\frac{\pi D_2 N}{60}$.

V_1 & V_2 = Absolute velocity of liquid at inlet & outlet.

V_{r_1} & V_{r_2} = Relative velocity of liquid at inlet & outlet.

V_{w_1} & V_{w_2} = Velocity of whirl at inlet and outlet.

V_{f_1} & V_{f_2} = velocity of flow at inlet & outlet.

α = Jet angle at inlet it is the angle made by absolute velocity (V_1) at inlet with the direction of motion of vane.

θ = vane angle at inlet, it is the angle made by the relative velocity (V_{r_1}) at inlet with the direction of motion of vane.

β & ϕ are the corresponding values at outlet. A centrifugal pump is just the reverse of a radially inward flow reaction turbine. Thus, workdone by centrifugal pump (or) impeller per second

workdone by centrifugal pump (or) impeller per second.

$\frac{\rho g a v}{g}$ $\frac{w a v}{g}$ = -workdone by radial inward flow reaction turbine per second.

$= -\rho a V_1 [V_{w_1} U_1 - V_{w_2} U_2] = -\rho Q (V_{w_1} U_1 - V_{w_2} U_2) = \rho Q (V_{w_2} U_2 - V_{w_1} U_1)$

$= \frac{wQ}{g} [V_{w_2} U_2 - V_{w_1} U_1] = \frac{wQ}{g} \times V_{w_2} U_2 = \rho Q (V_{w_2} U_2)$ $[\because V_{w_1} = 0]$.

work done by the impeller on the water per unit weight of water.

$= \frac{1}{g} [V_{w_2} U_2 - V_{w_1} U_1] = \frac{1}{g} [V_{w_2} U_2]$.

power output of the pump $= wQH_m = \frac{wQ V_{w_2} U_2}{g}$ watts.

Discharge = Area × velocity [Q (or) volume of water].

$Q = \pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2}$.

Definition of Heads:

1. Suction head (h_s): It is the vertical height of the centre line of the centrifugal pump above the water surface in the pump from which water is to be lifted. It is denoted by h_s.

2. Delivery head: The vertical distance between the centre line of the pump and the water surface in the tank to which water is delivered is known as delivery head. This is denoted by h_d.

3. Static head (H_s):- The sum of suction head and delivery head is known as static head. This is represented by H_s and is written as $H_s = h_s + h_d$.

4. Manometric head (H_m):- The manometric head is defined as the head against which a centrifugal pump has to work. It is denoted by 'H_m'. It is given by the loss of impeller to the water head in pump.

H_m = Head imparted by the loss of head.

$$= \frac{V_{w2} U_2}{g} - \text{Loss of head}.$$

$$= \frac{V_{w2} U_2}{g} \text{ (if loss of head is zero).}$$

Efficiencies of a centrifugal pump:-

In case of centrifugal pump, the power is transmitted from the shaft of the electric motor to the shaft of the pump and then to the impeller from the impeller, the power is given to the water. Thus power is decreasing from the shaft of the pump to the impeller and then to the water. Important efficiences of centrifugal pump are.

a. Manometric efficiency (η_{mano})
b. Mechanical efficiency η_m.
c. overall efficiency η_o.

Manometric efficiency (η_{mano}):-

The ratio of the manometric head developed by the pump to the head imparted by the impeller to the water is known as manometric efficiency.

$$\eta_{mano} = \frac{\text{Manometric Head}}{\text{Head imparted by impeller to water}} = \frac{(H_m)}{\frac{V_{w2} U_2}{g}} = \frac{g H_m}{V_{w2} U_2}.$$

Power at the impeller of the pump is more than power given to water at outlet of the pump. Ratio of the power given to the water at outlet of the pump to the power available at the impeller is known as manometric efficiency. power given to water at outlet of pump $\boxed{P = \frac{W H_m}{1000} \text{ kW}}$

$$\text{Power at the impeller} = \frac{\text{workdone by per second}}{1000} \text{ kW} = \frac{W}{g} \times \frac{V_{w2} U_2}{1000} \text{ kW}.$$

$$\eta_{mano} = \frac{\frac{W H_m}{1000}}{\frac{W}{g} \times \frac{V_{w2} U_2}{1000}} = \frac{g H_m}{V_{w2} U_2}.$$

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Mechanical Efficiency (η_m):

Ratio of power delivered by the impeller to the liquid to the power input to the pump shaft is known as mechanical efficiency.

$$\eta_m = \frac{\text{Power delivered by the impeller to liquid}}{\text{Power input to the pump shaft}}$$

Power at the impeller kw = $\frac{\text{workdone by impeller per second}}{1000} = \frac{w}{g} \times \frac{V_{w2}u_2}{1000}$

$$\eta_m = \frac{\frac{w}{g} \times \frac{V_{w2}u_2}{1000}}{SP} \quad (or) \quad \eta_m = \frac{\frac{wQ}{g}\left(\frac{V_{w2}u_2}{1000}\right)}{S.P.}$$

Overall efficiency (η_o):-

Ratio of power out put of the pump to the pump power input to the pump. Power out of the pump in pump = $\frac{\text{weight of water lifted} \times H_m}{1000}$

$$= \frac{wH_m}{1000}.$$

Power input to the pump = Power supplied by electric motor = SP. of the pump.

$$\eta_o = \left[\frac{\frac{wH_m}{1000}}{S.P}\right] \quad \boxed{\therefore \eta_o = \eta_{mano} \times \eta_m}.$$

Minimum starting speed for a centrifugal pump:-

If the pressure rise in the impeller is more than or equal to manometric head (H_m), the centrifugal pump will start deliverying water.

* when impeller is rotating, the water in contact with the impeller is also rotating and it is a forced vortex.
* For forced vortex, centrifugal head (or) head due to pressure rise in the impeller

$$= \frac{\omega^2 r_2^2}{2g} - \frac{\omega^2 r_1^2}{2g},$$

ωr_2 = Tangential velocity of impeller at outlet (u_2)

ωr_1 = Tangential velocity of impeller at inlet (u_1)

$\therefore$ Head rise due to pressure rise in impeller $= \frac{u_2^2}{2g} - \frac{u_1^2}{2g}$.

water will flow only when head rise will be equal (or) greater than H_m.

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} \geq H_m.$$

for minimum starting speed $\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = H_m$.

we have $\eta_{mano} = \frac{gH_m}{V_{w2}u_2} \Rightarrow H_m = \eta_{mano} \times \frac{V_{w2}u_2}{g}$.

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = \eta_{mano} \times \frac{V_{w2}u_2}{g}.$$

we have $u_1 = \frac{\pi D_1 N}{60}$ & $u_2 = \frac{\pi D_2 N}{60}$.

$$\left[\frac{\pi D_2 N}{60}\right]^2 \times \frac{1}{2g} - \frac{1}{2g}\left[\frac{\pi D_1 N}{60}\right]^2 = \eta_{mano} \times \frac{V_{w2}\pi D_2 N}{g \times 60}.$$

$$\frac{1}{2g}\left[\frac{\pi N}{60}\right]^{\cancel{2}}\left[D_2^2 - D_1^2\right] = \eta_{mano} \times \frac{V_{w2} \times \pi D_2 \cancel{N}}{60 \cancel{g}}$$

$$\therefore N_{min} = \frac{120 \times \eta_{mano} \times V_{w2} \times D_2}{\pi\left[D_2^2 - D_1^2\right]}.$$

Priming of centrifugal Pump:

When a centrifugal Pump is not running for some time the water present in the pump casing and suction pipe flows back to the sump and these spaces get filled with air. Now, when the pump motor is switched on and pump start running, the head developed equals $H = \frac{V_{w2} u_2}{g}$ m of air. Since (air << water) the head thus generation cannot produce spontaneously the vaccum required to start the pumping action. Accordingly the water cannot be sucked in along the suction pipe to reach the impeller. For making the pump deliver water there is need to make the casing, impeller and suction line free from air and fill these spaces with water, this process is called priming.

These priming of centrifugal pump is the operation of filling the suction pipe, casing and a portion of the delivery pipe completely from outside. Source with the liquid to be raised before, starting the pump to remove any air, gas (or) vapour from these parts of the pump. After proper priming and keeping the delivery valve closed, the pump is started. The shut-off head builts up and when the delivery valve is gradually valve is gradually. A continuous discharge then flows from the pump.

If a centrifugal pump is not primed before starting, air pockets, inside the impeller may give rise to vortices and cause discontinuity of flow further, dry running of the pump may result in rubbing and seizing of the wearing rings, and cause serious damage.

Methods of Priming:-

The following methods are adopted for the priming of centrifugal pump.

1. Manual priming. 2. Vaccum priming. 3. Self priming.

Manual priming:-

Small pumps are usually primed by pouring the liquid directly in the pump casing through a funnel, provided at the top of the casing. The air-vent provided in the casing is opened to facilitate the exit of the air. The flowing out of excess water through this vent indicator that the air has been completely displaced from the suction pipe and the pump casing and that the system is thoroughly filled with water. The air-vent is then closed and pump motor is switched on.

Vaccum priming:-

Large pumps are provided by excavating the casing and the suction pipe by a vaccum pump or by an ejector, the liquid is thus drawn up the suction pipe from the sump & the pump is filled with liquid.

Self priming:-

The internal construction of some pumps is such that special arrangements containing a supply of liquid are provided in the suction pipe due to which automatic priming of the pump occurs is known as "Self-priming pumps".

Specific speed:

Specific speed of a centrifugal pump is defined as the speed of a geometrically similar pump which would deliver one cubic meter of liquid per second against a head of one meter and is denoted by N_s.

$[N = N_s;$ under $H_m = 1m;$ $Q = 1m^3/s]$.

$Q =$ area $\times$ velocity of flow $= \pi D B V_f$.

$Q \propto D \times B \times V_f$ —— ①.

$D \propto B$ —— ②.

D = Diameter of the impeller of pump, B = width of impeller.

Sub ② in ① $\Rightarrow$ $Q \propto D^2 V_f$ $\left[\because K_f = \frac{V_f}{\sqrt{2gH_m}} \Rightarrow V_f = K_f \times \sqrt{2gH_m}\right]$

flow ratio

$\therefore V_f \propto \sqrt{H_m}$ —— ③.

Sub ③ in above eqn

$Q \propto D^2 \sqrt{H_m}$ —— ④.

we also know tangential velocity $u = \frac{\pi D N}{60}$.

$u \propto DN$

$u \propto \sqrt{H_m}$.

$\therefore \sqrt{H_m} \propto u \propto DN$.

$\therefore DN \propto \sqrt{H_m}$.

$D \propto \frac{\sqrt{H_m}}{N}$ —— ⑤.

⑤ in ④ $\Rightarrow$ $Q \propto \left[\frac{\sqrt{H_m}}{N}\right]^2 \sqrt{H_m}$. $\Rightarrow Q \propto \frac{H_m^{3/2}}{N^2}$.

$Q = C \cdot \frac{H_m^{3/2}}{N^2}$ —— ⑥ $[\because N = N_s;$ $Q = 1m^3/sec,$ $H_m = 1m]$.

$\Rightarrow 1 = C \frac{(1)^{3/2}}{N_s^2}$. $\boxed{\therefore N_s^2 = C}$.

Sub 'c' value in ⑥ $\Rightarrow$ $Q = N_s^2 \frac{H_m^{3/2}}{N^2}$, $\Rightarrow N_s^2 = \frac{QN^2}{H_m^{3/2}} = \frac{N\sqrt{Q}}{H_m^{3/4}}$

$N_s^2 = \frac{N^2 Q}{H_m^{3/2}}$. $\left[\because N_s = \sqrt{\frac{N^2 Q}{H_m^{3/2}}}\right]$.

$$\boxed{\therefore N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}}$$

Limitation of suction lift / maximum suction lift / maximum suction height:-

Free surface of liquid is at a depth of h_s below the pump axis. This liquid is flowing with a velocity of V_s in the suction pipe.

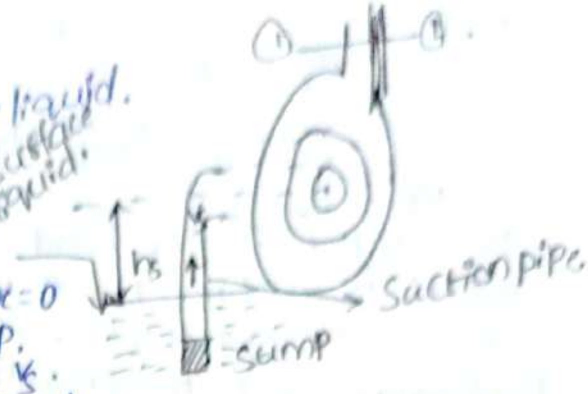
let; h_s = suction height (or) (lift).

Applying Bernoulli's equation at the free surface of the liquid in the sump and section ① in the suction pipe just at the inlet of the pump and taking free surface as the datum line.

$$\frac{P_a}{\rho g} + \frac{V_a^2}{2g} + Z_a = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 + h_L$$

$\frac{Pa}{\rho g} +$

Pa = Atmospheric pressure on the free surface of liquid.
Va = velocity of liquid at the free surface of liquid = 0.
Za = Height of free surface from datumline = 0
P_1 = Absolute pressure at the inlet of pump.
V_1 = velocity of liquid through suction pipe = V_s.
Z_1 = Height of Inlet of pump from datum line = h_s.
h_L = Loss of head in the foot valve, strainer and suction pipe = h_{fs}.



$$\frac{Pa}{\rho g} + 0 + 0 = \frac{P_1}{\rho g} + \frac{V_s^2}{2g} + h_s + h_{fs}.$$

$$\frac{Pa}{\rho g} = \frac{P_1}{\rho g} + \frac{V_s^2}{2g} + h_s + h_{fs}.$$

$$\frac{P_1}{\rho g} = \frac{Pa}{\rho g} - \left[\frac{V_s^2}{2g} + h_s + h_{fs}\right].$$

for maximum suction height pressure at inlet of the pump should not be less than vapour pressure of the liquid.

$$\frac{P_v}{\rho g} = \frac{Pa}{\rho g} - \left[\frac{V_s^2}{2g} + h_s + h_{fs}\right] \qquad \left[\because P_1 = P_v \text{ (vapour pressure)}\right]$$

$\frac{Pa}{\rho g}$ = Atmospheric pressure head = H_a.

$\frac{P_v}{\rho g}$ = vapour pressure head = H_v.

$$H_v = H_a - \left[\frac{V_s^2}{2g} + H_s + h_{fs}\right]$$

$$\boxed{h_s = H_a - H_v - \frac{V_s^2}{2g} - h_{fs}.}$$

The above equation represents the maximum suction lift. Suction height for any pump should not more than the above equation, and leads to vapourisation of the liquid at the inlet and finally leads to cavitation if the height is more than the above given equation.

Net positive suction Head [NPSH] :- (2M)

NPSH is defined as absolute pressure head at the inlet to the pump minus the vapour pressure head plus velocity head.

NPSH = Absolute pressure head − vapour pressure head + velocity head.

$$NPSH = \frac{P_1}{\rho g} - \left(\frac{P_v}{\rho g} + \frac{V_s^2}{2g}\right) \quad — (1).$$

But from maximum suction height we know,

$$\frac{P_1}{\rho g} = \frac{Pa}{\rho g} - \left[\frac{V_s^2}{2g} + h_s + h_{fs}\right] \quad — (2)$$

(2) in (1) $\Rightarrow NPSH = \frac{Pa}{\rho g} - \frac{V_s^2}{2g} - h_s - h_{fs} - \frac{P_v}{\rho g} + \frac{V_s^2}{2g}.$

$NPSH = \frac{P_a}{\rho g} - \frac{P_v}{\rho g} - h_s - h_{fs} = H_a - H_v - h_s - h_{fs}$

$= [(H_a - h_s - h_{fs}] - H_v]$

$H_v = H_a - h_s - h_{fs}$.

H_v = Total Suction head; NPSH = Total suction head.

NPSH is used in pump industry while manufacturing to have cavitation free centrifugal pump NPSH should be greater than required.

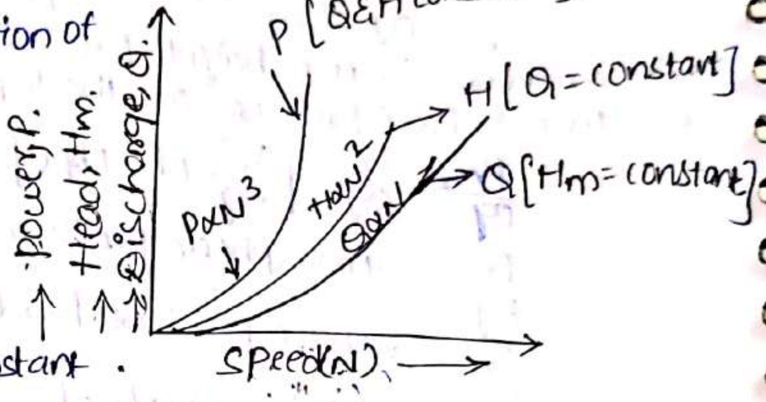
Performance and characteristic curves:-

Based on number of tests, characteristics curves are drawn for centrifugal pump, which are used to determines the behaviour and performance of the pump under working conditions. characteristic curves for pumps are

1. Main characteristics curves.
2. Operating characteristics curves
3. constant efficiency & Muschel curves.

Main Characteristic curves:-

A curve drawn between variation of (Hm), power (P), discharge (Q) versus Speed (N) is known as main characteristic curves.



* for curve Hm vs N – Q is constant.
* for curve Q vs N – Hm is constant.
* for curve P vs N – Hm & Q are constant.
* for Hm vs N [Q = constant].

$\frac{\sqrt{Hm}}{DN}$ = constant (or) $Hm \propto N^2$.

∴ curve is parabolic Since $H \propto N^2$.

* for P vs N [Q & H are constant]

$\frac{P}{D^5 N^3}$ = constant [D = pipe dia].

$P \propto N^3$.

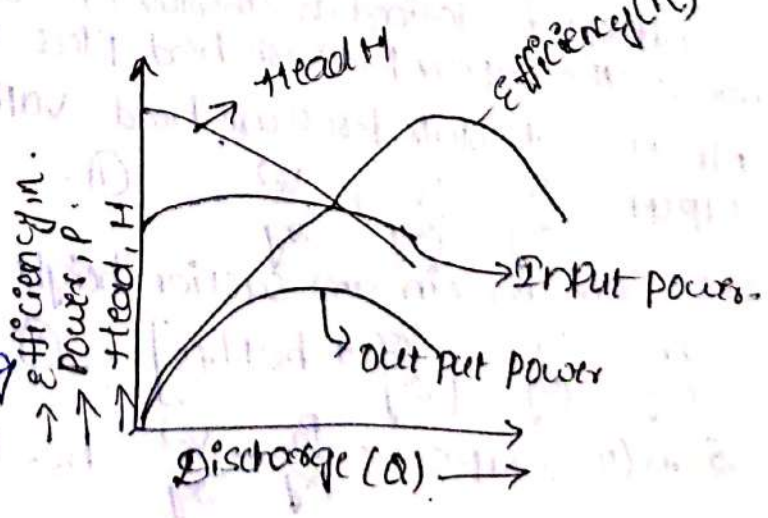
∴ curve is cubic curve.

* for Q vs N, $\frac{Q}{D^3 N}$ = constant ⇒ $Q \propto N$

∴ curve is straight line

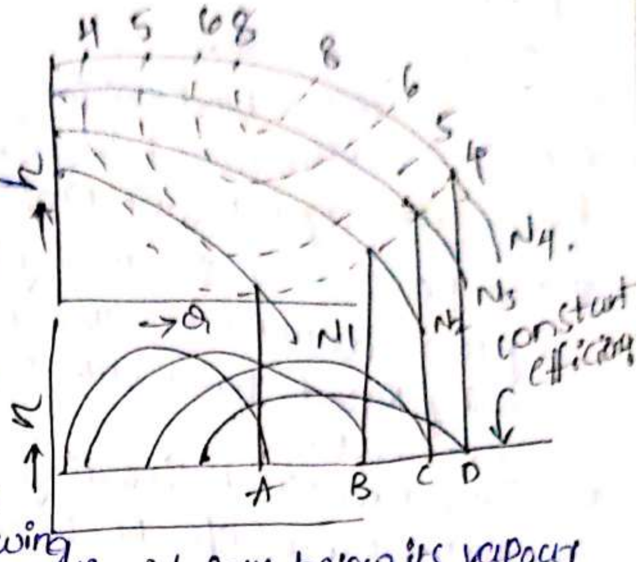
Operating Characteristic curves:-

By placing speed as constant, variation of head, power, efficiency w.r.to discharge and curves are to be drawn which gives the operating characteristic curves of pump.



* Input power doesn't pass from origin, because at zero discharge some power is required to overcome mechanical losses.
* Head will be maximum when $Q=0$
* $Q=0$, $\eta=0$ hence pass from origin output power will pass from origin

Constant efficiency curves:



Head vs discharge and efficiency vs discharge curves are to be drawn for different speeds.

* By combining above curves constant efficiency.
* Iso-efficiency curves are obtained.

Cavitation:- (2M)

Cavitation is defined as the phenomenon of formation of vapour bubbles of a flowing liquid in a region where the pressure of the liquid falls below its vapour pressure and sudden collapsing of these values vapour bubbles in a region of higher pressure.

Formation of vapour bubble of the flowing liquid take place only whenever the pressure in any region falls below vapour pressure when the pressure of the flowing liquid is less than its vapour pressure, the liquid starts boiling and vapour bubbles are formed. These vapour bubbles are carried along with the flowing liquid to higher pressure zones where the vapours condense and bubbles collapse due to sudden collapsing of the bubbles on the metallic surfaces, high pressure is produced and metallic surfaces are high local stresses, thus cavities are formed on the metallic surface and also considerable noise and vibrations are produced.

In order to determine whether cavitation will occur in any portion of the section side of the pump. Thoma's cavitation factor (σ) is used. Thoma's cavitation factor for centrifugal pump is given by.

$$\sigma = \frac{H_a - H_s - H_v}{H_{mano}} = \frac{H_{sv}}{H_{mano}}$$

where;

H_a = Atmospheric pressure express in 'm' of water head.

H_s = Total suction head $\left[= h_s + h_{fs} + \frac{V_s^2}{2g}\right]$.

H_v = vapour pressure express in 'm' of water head.

H_{sv} = Net positive suction head (NPSH) &

H_{mano} = manometric head.

Note:- * The intensity of cavitation increases with the decrease in value of NPSH.

* The cavitation of pump can be noted by a sudden drop in efficiency, head & power requirement.

* Cavitation will occur if value of σ is less than critical value (σ_c) at which cavitation just begins.

Effects of cavitation:

* pitting and erosion of surface due to continuous hammering action of collapsing bubbles.
* Sudden drop in head, efficiency and power delivers to the fluid.
* Noise and vibrations introduced by the collapse of bubbles.

Precautions against cavitation:

* The pressure of the flowing liquid in any part of the hydraulic system should not be allowed to fall below its vapour pressure of the flowing liquid, then absolute pressure head should not be below 2.5 m of water.
* Special materials & coatings such as aluminium, bronze & stainless steel materials should be used.

Multi-Stage centrifugal pumps:-

If a centrifugal pump consists of two & more identical impellers mounted on the same shaft & on different shafts, the pump is called as multi-stage centrifugal pump. If the impellers are arranged on the same shaft then it is called series arrangement and if the impellers are mounted on different shafts, then it is called parallel arrangement.

Functions of multistage pump:-

1. To produce a high head by series arrangement.
2. To discharge a large quantity of liquid by parallel arrangement.

Pump in series / High head:-

For obtaining a high head, a no. of identical impellers are mounted in series & on the same shaft and enclosed in the same casing. The discharge from impeller-1 passes through a guided passage and enters the impeller-2. At the outlet of impeller-2, the pressure of water will be more than the pressure of water at outlet of impeller-1. Thus, if more no. of impeller are mounted on the same shaft the pressure at outlet will be increased further.

let;

n = No. of identical impellers mounted on the same shaft.

H_m = Head developed by each impeller.

∴ Total head devoloped $H_{total} = n \times H_m$.

* The series arrangement is employed for delivering a relatively small quantity of liquid against very high heads.

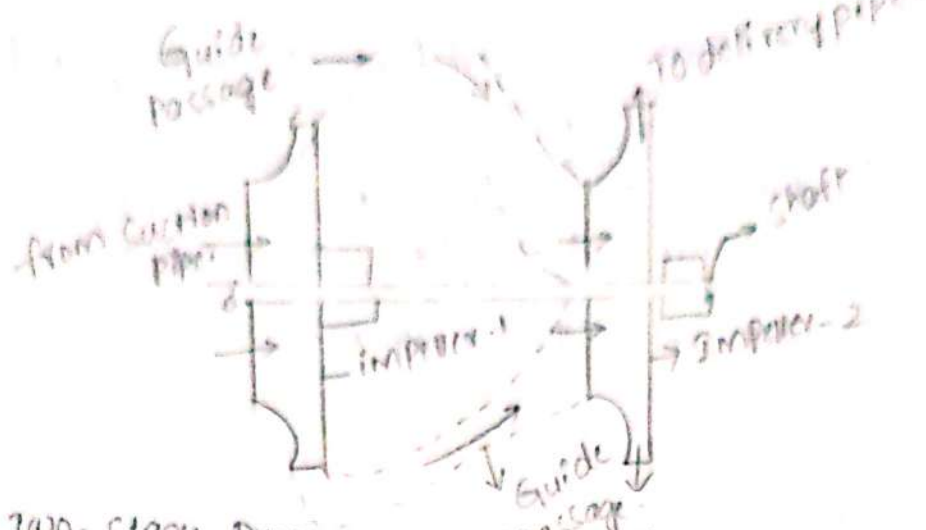


fig:- Two-stage pump impellers in series.

Pumps in parallel Discharge High:-

For obtaining a large quantity of liquid, two or more pumps are arranged in parallel. In this arrangement each of the pump working separately lifts. The liquid from a common sump & delivers it to a common collecting pipe, through which it is carried to the required height.

In this arrangement each of the pump working separately lifts the liquid from a common sump and delivers it to a common collecting pipe, through which it is carried to the required height.

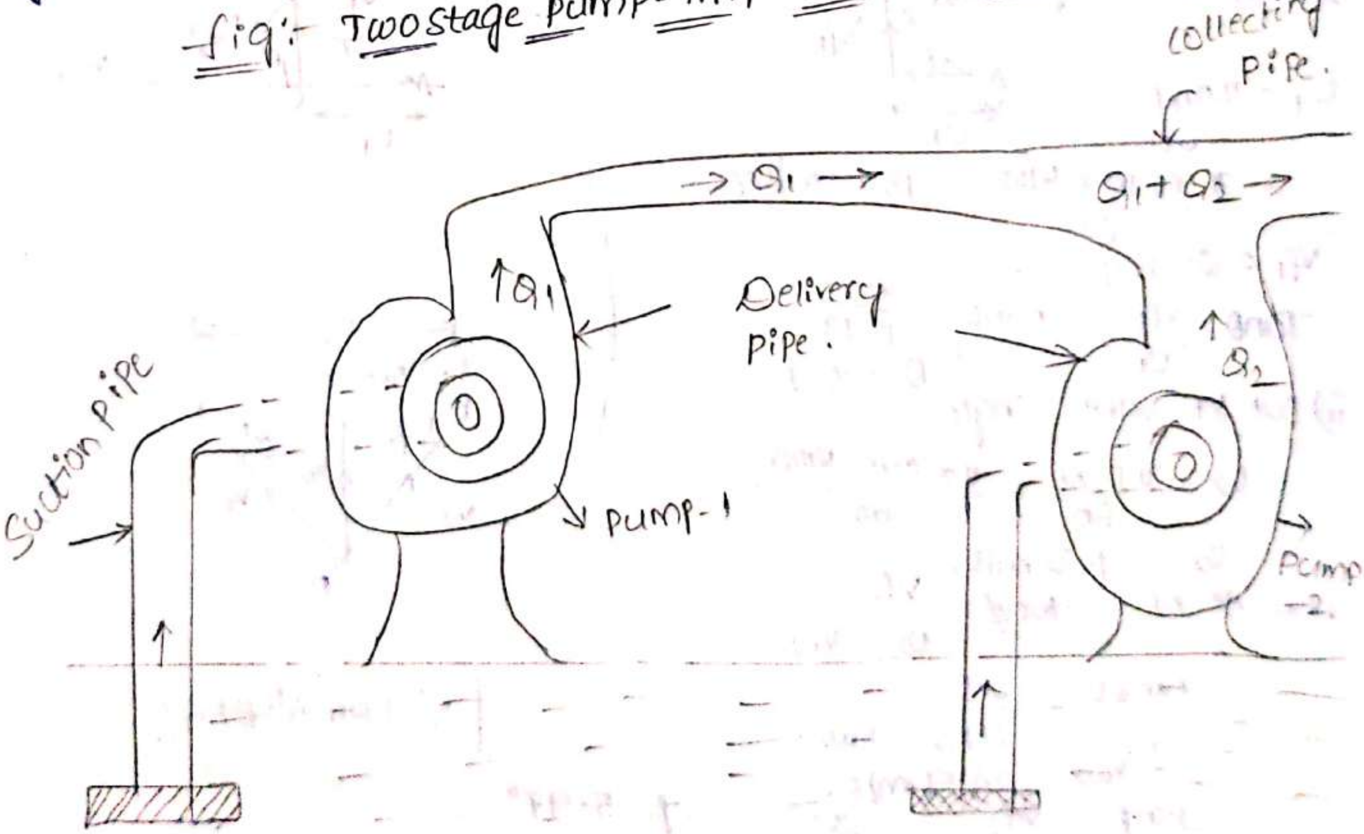
let n = No. of identical pumps arranged in parallel

Q = Total discharge from one pump.

$\therefore$ Total discharge $Q_{total} = n \times Q$.

* parallel arrangement is employed for delivering a large quantity of liquid against a relatively small head.

fig:- Two stage pump- impellers in parallel.

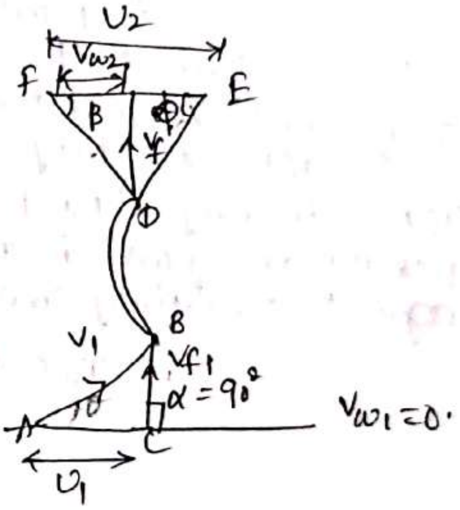


Centrifugal Pumps:

1. These pumps can run at higher speeds and hence the discharge is high.
2. It can be used for lifting highly viscous liquids.
3. The wear and tear is less because of less moving parts.
4. Cost of centrifugal pumps is less compared to reciprocating pump.
5. It needs smaller floor area and installation cost is low.
6. The flow is uniform.
7. It's operation is smooth and without much noise.
8. It's maintenance cost is low.
9. efficiency is high.
10. Requires priming.

Problems on velocity Triangles:-

① The impeller of a centrifugal pump has an external dia of 400mm and internal dia of 180mm runs at 1400 rpm assuming velocity of flow through the impeller is radial and constant at 2.5 m/se & the vanes are set back at an angle of 25°c at the exit. Determine i) Inlet vane angle
ii) Angle made by absolute velocity of water at exit.
iii) workdone per newton of water.



Sol: Given,

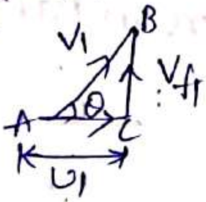
$D_2 = 400mm = 0.4m$

$D_1 = 180mm = 0.18m.$

$U_1 = 1400 \, rpm.$

$V_{f1} = V_{f2} = 2.5 \, m/sec$ & $\phi = 25^0 c.$

i) Inlet vane angle



$U_1 = \frac{\pi D_1 N}{60}$

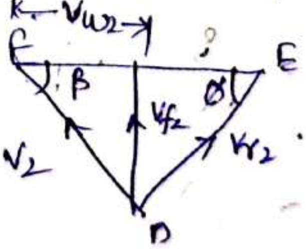
$= \frac{\pi \times 0.18 \times 1400}{60} = 13.19 \, m/sec.$

$V_{f1} = 2.5 \, m/sec.$

$Tan\theta = \frac{V_{f1}}{U_1} \Rightarrow Tan\theta = \frac{2.5}{13.19}.$

$\therefore \theta = 10.7$

ii) Out let vane angle.



$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 1400}{60}.$

$U_2 = 29.32 \, m/s.$

from Δ^{le} DEG $\tan\phi = \frac{V_{f2}}{U_2 - V_{w2}}$

$Tan 25 = \frac{2.5}{29.32 - V_{w2}}$ [$\because$ from Δ^{le} DFG].

$\therefore V_{w2} = 23.9 \, m/s.$

$Tan\beta = \frac{V_{f2}}{V_{w2}} = \frac{2.5}{23.9}$ $\therefore \beta = 5.97°$

iii) work done per sec $= \rho a v [V_{w_2} U_2 - V_{w_1} U_1]$

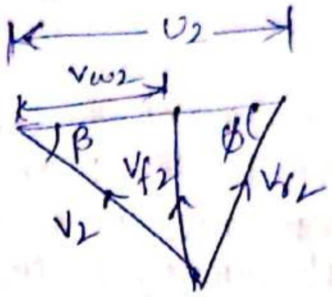
$$\frac{W.D/sec}{N} = \frac{\rho a v [V_{w_2} U_2 - V_{w_1} U_1]}{\rho a v \times g} \quad [V_{w_1} = 0]$$

$$= \frac{V_{w_2} U_2}{g} = \frac{23.7 \times 29.32}{9.81}$$

$$\frac{W.D/sec}{N} = 71.43 \ \frac{N \cdot m/sec}{N}$$

(2) centrifugal pump with radial in flow delivery discharge $0.08 m^3/sec$ of water against a head of 40m if the outer diameter of impeller is 300mm & its width at outlet periphery is 12.5mm. Find the vane at the exit. The speed of the pump is 1500 rpm and the manometric efficiency is 80%.

Sol: Given,



$Q = 0.08 \ m^3/sec$.

$H_m = 40m$.

$D_2 = 300mm = 0.3m$

$B_2 = 12.5mm = 0.0125m$.

$N = 1500 \ rpm$.

$\eta_{mano} = 80\% = 0.8$

$$\tan\phi = \frac{V_{f2}}{U_2 - V_{w2}}$$

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.3 \times 1500}{60} = 23.56 m/sec.$$

$Q = \pi D_2 B_2 V_{f2} \Rightarrow 0.08 = \pi \times 0.3 \times 0.0125 \times V_{f2}$

$V_{f2} = 6.79 \ m/sec$.

$$\eta_{man} = \frac{g H_m}{V_{w2} U_2} \Rightarrow 0.8 = \frac{9.81 \times 40}{V_{w2} \times 23.56}$$

$V_{w2} = 20.82 \ m/sec$.

$$\tan\phi = \frac{6.79}{23.56 - 20.82}$$

$\therefore \phi = 68.02$.

(3) A centrifugal pump is required to deliver $0.048 m^3/sec$ of water to a height of 24m, through a 150mm diameter pipe and 120 long. If the overall efficiency of the pump is 75% and co-efficient of friction $f = 0.01$ for the pipeline find the power required to drive the pump.

Sol: Given,

$Q = 0.048 \ m^3/se$; $H_s = 24m$; $D = 150mm = 0.15m$; $L = 120m$

$\eta_o = 75\% = 0.75$, $f = 0.01$.

$$\eta_{overall} = \frac{\text{Power delivered by Pump}}{\text{Power given by shaft}} = \frac{W Q H_m}{P}$$

$$0.75 = \frac{9810 \times 0.048 \times H_m}{P}$$

$$H_m = H_s + (h_{fs} + h_{fd}) + \frac{V_d^2}{2g}$$

$H_s = 24m$

$\left[h_{fs} + h_{fd}\right] = h_f = \frac{4fLV^2}{2gD}$

$V = \frac{Q}{A} = \frac{0.048}{\frac{\pi}{4}\times(0.15)^2} = \boxed{2.72\ m/sec = V}$

$* \ h_f = \frac{4\times0.01\times120\times(2.72)^2}{2\times9.81\times0.15} = 12.06m.$

$* \ \frac{V_d^2}{2g} = \frac{(2.72)^2}{2\times9.81} = 0.37m$

$H_m = 24 + 12.06 + 0.37 = 36.43m.$

$P = \frac{9810\times0.048\times36.43}{0.75}.$

$P = 22.87KN.$

(4) The impeller of the centrifugal pump has 1.2m out side diameter it is used to lift 1800lit/sec of water against a head of 6m. Its vanes makes an angle of 150° to the direction of motion at outlet and runs at 200rpm. If the velocity of flow at outlet is 2.5m/sec and the flow is radial. Find the manometric efficiency also. Find the lowest speed to start the pump if the diameter at the inlet is half of the diameter at outlet.

Sol: Given,

$D_2 = 1.2m$

$Q = 1800\ lits/sec = 1.8\ m^3/sec.$

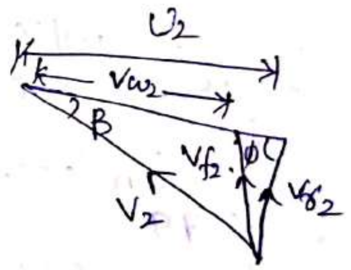
$H_m = 6m$

$N = 200rpm.$

$V_{f2} = 2.5m/sec.$

$\phi = 150°.$

Manometric efficiency $\eta_{mano} = \frac{gH_m}{V_{w2}U_2}$



$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi\times1.2\times200}{60} = 12.57\ m/sec.$

$\tan\phi = \frac{V_{f2}}{U_2 - V_{w2}}.$

$\tan150° = \frac{2.5}{12.57 - V_{w2}}.$

$V_{w2} = 16.9\ m/sec$

$\therefore \eta_{mano} = \frac{9.81\times6}{16.9\times12.57} = 0.28.$